The Macroeconomic Dynamics of Labor Market Policies*

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Abstract

We develop a dynamic macroeconomic framework with worker heterogeneity, monopsony power, and putty-clay adjustment frictions in order to study the distributional impact of labor market policies in the short and long run. Our model helps reconcile the tension between low short-run and high long-run elasticities of substitution across inputs of production. We use the model to assess the labor market effects of redistributive policies such as the federal minimum wage and the Earned Income Tax Credit (EITC). A key result of our analysis is that measuring the welfare impact of these policies requires taking into account the entire time path of the responses they induce. For instance, either increasing the minimum wage or expanding the EITC makes low-wage workers better off in the short run. However, the effects of the two policies can substantially differ in the long run. At longer horizons, both *small* minimum wage increases and EITC expansions of any size continue to make low-wage workers better off, whereas sufficiently *large* minimum wage increases adversely impact such workers. We find that combining either the EITC or the overall tax and transfer system with moderate increases in the minimum wage better supports the income and welfare of low-wage workers than either policy does in isolation, because doing so more effectively offsets firms' monopsony power. We conclude by discussing the conditions under which empirical estimates of the short-run effects of labor market policies are informative about their ultimate long-run effects through the lens of our model.

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1 Introduction

The high degree of wage inequality in the United States, coupled with increasing evidence of firm monopsony power in labor markets, has renewed interest in redistributive policies such as the minimum wage and the Earned Income Tax Credit (EITC). Although these policies are just meant to support the income of low-wage workers, they also affect the relative prices of these workers' services, by making them more expensive (the minimum wage) or less expensive (the EITC) to employ. Hence, the ultimate effects of these policies on employment, labor income, and welfare critically depend on firms' ability to substitute across workers. Incorporating the dynamics of such a margin of adjustment is important given that the estimates of this elasticity of substitution greatly vary with the time horizon considered. In particular, a large literature on the minimum wage has documented small employment effects of increases in the minimum wage in the year or two after an increase is introduced, suggesting a fairly small short-run elasticity of substitution among workers.[1] By contrast, the literature studying long-run changes in the distribution of skills among workers and the skill premium has consistently found a fairly large longer-run elasticity of substitution.[2]

Our starting point in this paper is to take these dynamics of labor-labor substitution seriously in evaluating policies. We do so by developing a macroeconomic framework to assess the distributional impact of labor market policies over time and disciplining it to be consistent with long-run features of U.S. labor markets. We then use this framework to study the effects of the minimum wage, the EITC, and the overall U.S. tax and transfer system. The key theme of our analysis is that accurately evaluating the impact on employment, labor income, and welfare of redistributive policies requires taking the entire time path of their effects into account. Intuitively, employment effects are much smaller in the short run, when the elasticity of substitution among workers is low, than in the long run, when this elasticity is high. These dynamics, for instance, imply that in response to the EITC, regardless of its size, the positive short-run effects on the income of low-wage workers, whose employment is subsidized, become even larger in the long run, as firms increasingly employ more of these workers. However, in response to a large minimum wage, the positive effects on the income of low-wage workers in the short run are eroded over time, since firms eventually substitute away from them. As a result, the welfare impact of these policies, which is determined by the entire dynamic

[1] See the classic work by Brown (1999) and Card and Krueger (2015) and, recently, Neumark and Shirley (2022).

[2] Prominent studies include Katz and Murphy (1992), which estimates an elasticity of substitution *across* workers of different education groups of about 1.5, and Card and Lemieux (2001), which finds an elasticity of substitution among workers *within* an education group between 4 and 6. In a similar vein, Chetty (2012) highlights how worker and firm adjustment costs imply that shorter-run labor supply elasticities are smaller than longer-run ones.

path of their effects, cannot be inferred from evidence on their short-run effects alone.

Our framework combines three ingredients that we argue are necessary to evaluate the distributional effects of policies that affect wage setting in both the short and the long run. First, in order to assess the distributional implications of such policies, it is essential to account for the dispersion in wages both within and across education groups. We use data from the 2017-2019 American Community Survey to measure the distribution of wages within and across education groups. A primary motivation for including both of these forms of worker heterogeneity is that within-education-group wage dispersion exceeds between-education-group wage dispersion by an order of magnitude: the wages in the highest decile of the wage distribution of non-college workers are about seven times larger than those in the lowest decile. By contrast, the ratio of the mean wages of college-educated workers to those of non-college-educated workers is less than two.[3] Bonhomme et al. (2022) provides recent evidence confirming that the dominant source of the variation in wages across workers is worker heterogeneity.[4] Second, we allow for imperfect long-run input substitutability through complementarity in production of the standard CES form, together with putty-clay frictions, which parsimoniously lead to a rich short-run and long-run dynamics of input adjustment.[5] In particular, these frictions yield short-run elasticities of employment with respect to any change in policy that are close to zero, because workers who operate existing capital are highly complementary to each other in the short run. Third, we account for firm monopsony power in the labor market and so capture the idea, dating back to Robinson (1933), that policies may not only redistribute income but also help correct market inefficiencies. We capture this insight in a modern framework by formalizing the notion of monopsony power within a search model of the labor market, which avoids any ad-hoc rationing rules when policy-mandated constraints, such as the minimum wage, bind.[6]

Our quantitative framework yields four key insights, which underscore the importance of accounting for the dynamic impact of labor market policies. First, a minimum wage or EITC policy makes low-wage workers unambiguously better off in the first few years after either policy is in-

[3]Within-education-group wage heterogeneity is critical when studying policies like the minimum wage and the EITC. For example, the median wage for both college and non-college workers in 2017 exceeded $15 per hour implying that the representative individual from both groups would not be bound by a $15 federal minimum wage. However, 40% of non-college workers earn a wage lower than $15.

[4]Applying their proposed fixed-effects and random-effects bias-correction procedures, these authors find that only 5% to 6% of the overall variance of log earnings is due to firm effects. They also report that no more of 15% of all earnings variation is due to sorting, that is, the covariance between worker and firm effects.

[5]We follow a procedure analogous to that in Katz and Murphy (1992) and Card and Lemieux (2001) to pin down the parameters of our long-run production function.

[6]We discipline the degree of firms' monopsony power so as to match the recent estimates of wage markdowns in Seegmiller (2021), Lamadon, Mogstad and Setzler (2022), Berger, Herkenhoff and Mongey (2022a), and Yeh, Macaluso and Hershbein (2022), which document that workers are paid between 65% and 85% of their marginal products.

troduced. This result occurs because the after-tax wages of low-wage workers immediately increase in response, but their employment only slowly adjusts due to putty-clay frictions, which require firms to adapt their capital stock to the new desired mix of workers. Because of this feature, our framework is consistent with a large empirical literature that documents small employment effects of the minimum wage in the short run.

Second, our framework implies that the dynamic effects of the minimum wage and the EITC can substantially differ at longer horizons. In particular, the long-run effects of the minimum wage critically depend on its size. For small to moderate increases in the minimum wage, the short- and long-run effects on employment and labor income are of the *same* sign. The wages of initially low-wage workers immediately increase and over time their employment increases as well, since the monopsony distortions that depress their wages and employment are offset by the minimum wage increase. As firms progressively adjust their input use towards low-wage workers, such minimum wage increases have even more beneficial effects on these workers in the long run than in the short run. However, the short- and long-run effects of large increases in the minimum wage on the employment and labor income of low-wage workers are of the *opposite* sign, because firms have an incentive to substitute away from these workers whenever a new minimum raises wages well above the efficient level. Hence, large minimum wage increases can have negative effects on the employment, labor income, and welfare of low-wage workers in the long run.

Differently from a minimum wage policy, the short- and long-run effects of an EITC policy always work in the *same* direction. The reason is that under the EITC, the government pays the marginal cost of increasing the wages of low-productivity workers and, hence, subsidizes their employment, whereas under the minimum wage, firms pay this cost. As a result, the EITC reduces monopsony distortions without creating an incentive for firms to substitute away from lower-productivity workers. Indeed, because of the employment subsidy that the EITC entails, firms find it optimal to substitute towards these workers. Putty-clay frictions dampen the positive effects of the EITC on employment in the short run, because it takes time for firms to expand their employment of low-wage workers by investing in new capital that is more intensive in the use of their labor services. Hence, the short-run benefits of the EITC are smaller than their ultimate long-run benefits. The size of the EITC only matters in the sense that larger subsidies induce firms to shift their input mix towards lower-wage workers even more in the long run. Overall, our comparison of the minimum wage and the EITC illustrates how the dynamic impact of labor market policies critically depends on whether such policies strengthen or weaken firms' incentives to adjust their production mix towards the targeted workers.

Our third key insight is that to assess the overall impact of labor market policies on welfare, the entire dynamics of their effects at different horizons need to be taken into account. When we do so in our quantitative model, we find that *all* non-college workers weakly benefit from a small to moderate increase in the minimum wage between \$1 and \$3 per hour, especially those earning less than the new minimum. Interestingly, even for a large increase to \$15 per hour, nearly 85% of non-college workers initially earning less than \$15 benefit from such a policy. Intuitively, for large increases as well, the benefits of higher wages and the slow adjustment of employment in the short run outweigh the larger employment losses in the long run for most low-wage workers. For the EITC, in contrast, essentially all workers benefit in both the short and the long run, but benefits are smaller at shorter horizons, regardless of the size of the EITC. Although both policies are beneficial overall to most low-wage workers in the short to the intermediate run, welfare gains are much larger under a (budget-equivalent) EITC than under the minimum wage. Again, this result stems from the fact that a large increase in the minimum wage, unlike a large expansion of the EITC, creates an incentive for firms to substitute away from the newly more expensive low-productivity workers in the long run. Yet, combining the EITC or, more generally, the overall tax and transfer system with moderate minimum wage increases is more beneficial than either policy alone for low-productivity workers in the long run, because it provides a more effective way to offset firms' monopsony power.

Our final insight is that the degree of persistence of policies, intended or perceived, is crucial for their impact. For instance, the short- and long-run effects of minimum wage increases are very different depending on whether the increase is permanent in real terms, as we have assumed so far, or in nominal terms. In the latter case—which pertains to most changes in the federal minimum wage in the United States—the real effective minimum wage decreases over time due to inflation and aggregate productivity growth. Since the real value of the minimum wage is eroded over time, the long-run real effects of an increase in it are small regardless of the size of the nominal increase—of course up to some level, as firms will eventually shut down if the minimum wage becomes too large. The reason for this difference in results is that given our putty-clay adjustment frictions, firms do not find it profitable to adjust their input mix in response to temporary changes in the real minimum wage. Indeed, if the minimum wage is expected to be less and less binding on lower-productivity workers in the future, firms do not have any incentive to pay the cost associated with adjusting their input mix away from such workers in the short run and then back towards them in the long run. Therefore, if the minimum wage is set in nominal terms, large increases are highly beneficial for lower-productivity workers in the short run at virtually no long-run cost.

We conclude the paper with a discussion of the insights that our framework provides for interpreting, and possibly informing, empirical research. First, our framework offers a rationale for the large heterogeneity in the estimated employment effects of the minimum wage documented in the literature. Our framework can lead to either small positive or small negative employment effects in the short run in response to moderate minimum wage increases depending on *i*) the size of the increase considered; *ii*) the strength of firms' monopsony power in the relevant labor market; *iii*) the degree of input substitutability in the long run; and *iv*) the extent to which the increase is believed to be permanent in real terms. As a result, any estimate of the labor market effects of an increase in the minimum wage must implicitly take a stand on the level of firms' monopsony power, the degree of input substitution, and firms' beliefs about the persistence of the increase. These forces almost certainly differ across different labor markets, across regions within the same country, between regional and national levels of a given country, and across countries, which suggests caution when comparing effects across different settings. Second, our framework provides some guidance on when it is appropriate to extrapolate short-run estimates of the impact of a given policy to infer its potential long-run impact. We demonstrate that for small increases in the minimum wage or for expansions of the EITC of any size, their short-run labor market effects are quite informative about their ultimate long-run effects without the need for any additional assumptions. However, this is not the case for large increases in the minimum wage. As we show, forecasting the long-run effects of large minimum wage increases from short-run evidence on their effects requires explicitly accounting for the key features of an economy that we have emphasized throughout.

Related Literature. Our paper provides a rich framework for the evaluation of policies that may have potentially conflicting effects across workers with different characteristics and over different time horizons. Our approach follows in the footsteps of an established macroeconomic literature that has investigated the general-equilibrium impact of specific income-support policies such as the minimum wage (Lise, Meghir and Robin (2016) and Engbom and Moser (2021)) as well as that of broad large-scale redistributive policies such as the U.S. tax and transfer system (Heathcote, Storesletten and Violante (2017)). We add to this literature by emphasizing the importance of accounting for the dynamic effects of policies, which may entail qualitatively distinct distributional trade-offs across the short and the long run.

Our paper offers some insights related to those in Sorkin (2015) and Aaronson et al. (2018), who use a variant of the standard putty-clay capital setup to argue that the effect of the minimum wage on employment is smaller in the short run than in the long run. Specifically, these papers allow for only an extensive margin of choice on the part of firms with respect to whether they operate at full capacity or completely shut down. In contrast to these papers, which focus solely on the entry and exit margin of firms with limited worker heterogeneity, we also allow for the intensive margins of capital and labor substitution and of labor-labor substitution in a model with rich worker heterogeneity. We discipline these margins by using well-known measures of the long-run elasticities of substitution among inputs in the literature, as in Katz and Murphy (1992) and Card and Lemieux (2001). The labor-labor elasticities in particular turn out to be crucial for the welfare impact of policies. In addition, we *i*) incorporate firm monopsony power in order to allow for the possibility that the minimum wage improves on equilibrium allocations; *ii*) study the EITC and the overall tax and transfer system in addition to the minimum wage; and *iii*) use a general equilibrium model to examine large changes in policy that impact the entire economy.

Our framework also builds on a long line of research that has evaluated the impact of labor market policies through the lens of frictional models of the labor market. See, in particular, Eckstein and Wolpin (1990), Flinn (2006), Ahn, Arcidiacono and Wessels (2011), Engbom and Moser (2021), and Drechsel-Grau (2022). We differ from this literature in that we augment the standard search framework with monopsonistic competition among firms and assess the distributional effects of labor market policies in both the short and the long run.

Finally, our model of monopsonistic competition is the natural labor market analog of a model of monopolistic competition in the goods market, extended to a search setting. In our model, firms' monopsony power arises from the imperfect substitutability of jobs across firms in workers' preferences, as in Berger, Herkenhoff and Mongey (2022a). In contemporaneous work, Berger, Herkenhoff and Mongey (2022b) uses this setup and adapts the Atkeson and Burstein (2008)'s model of Cournot competition in the goods market to a labor market setting so as to pursue a normative analysis of the long-run optimal level of the minimum wage. Our paper differs from Berger, Herkenhoff and Mongey (2022b) in that we emphasize that the transition dynamics induced by any policy changes is critically important for evaluating their welfare impact—especially as short-run impacts often have the opposite sign of long-run ones. We also demonstrate that it is important to account for the rich within-education-group worker heterogeneity in the data in order to capture the key margin of labor-labor substitution within education groups, as emphasized by Card and Lemieux (2001). By the same token, our model abstracts from the rich oligopsonistic market structure studied in Berger, Herkenhoff and Mongey (2022b). Thus, we view the two papers as complementary.

2 Model

Our model integrates three key features. First, we incorporate the notion of firm monopsony power in labor markets by allowing workers to view jobs at different firms as imperfectly substitutable with each other. When this is the case, any policy that reduces the associated monopsony distortions can potentially increase not only labor income but also employment and labor market participation.

Second, we allow for rich heterogeneity in worker productivity, both within and across education groups, to comprehensively explore the distributional impact of policies. We capture it within a production structure consistent with the long-run evidence on the elasticity of substitution across inputs. For example, our model matches both the long-run elasticity of substitution between college and non-college workers estimated by Katz and Murphy (1992) and the elasticity of substitution across workers with different experience or ability *within* an education group estimated by Card and Lemieux (2001). We also let firms substitute between labor and capital.

Finally, to guarantee that our technology is consistent with the evidence of low input substitutability in the short run, we assume that firms operate technologies of the putty-clay type. Specifically, we capture in a parsimonious way the notion that adjusting the ratio of *any* inputs is costly for firms in the short run by assuming that the labor intensity of each unit of capital is irreversible once that capital is installed. We view this setup as a tractable way of capturing the spirit of a rich set of adjustment costs when firms decide to alter the mix of any of their inputs. In our quantitative exercises, we show how the presence of putty-clay capital slows down the transition of the economy to a new steady state after any change in policy. In particular, putty-clay capital helps the model reproduce the well-documented feature that employment responses to increases in the minimum wage tend to be fairly muted in the short run, but can be larger in the long run, as consistent with a large literature on long-run input substitution patterns.

2.1 Preferences, Production, and Labor Market Matching

We consider an infinite-horizon economy in discrete time populated by consumers and firms. We describe next consumers, production, labor market matching, and the timing of events. See Appendix A and the online appendix for omitted details.

Consumers: Heterogeneity and Preferences. Consumers are heterogeneous in two dimensions. First, they differ in their *education level* $g \in \{\ell, h\}$, where ℓ denotes the group of low-educated consumers (those with less than a bachelor's degree) and h denotes the group of high-educated consumers (those with a bachelor's degree or more). Second, within each group g, consumers are characterized by an *ability* or *productivity level* z drawn from education-specific discrete distributions. Ability differences among consumers allow us to match the observed wage distribution for each education group. We index a consumer by i, which denotes both a consumer's education group and ability level so that $i \in I \equiv I_\ell \cup I_h$, where $I_\ell = \{i|\ z_i \in \{z_{\ell 1}, \ldots, z_{\ell M}\}\}$ represents the set of abilities of low-educated consumers and $I_h = \{i|\ z_i \in \{z_{h1}, \ldots, z_{hM}\}\}$ represents the set of abilities of high-educated consumers. As shorthand, we let $i = (g, z_{gi})$ denote an education-ability pair. The economy consists of a measure μ_i of families of each type i per firm.[7]

Each type of family is composed of a large number of members of the same education group and ability level. Risk sharing within each such family implies that each member of a household of type i consumes the same amount of goods at each date t. Note that here we summarize the insurance arrangements in the economy through a representative family construct, as in Merz (1995) and Andolfatto (1996), which implies perfect sharing of the idiosyncratic risk leading to income losses, which arises from both unsuccessful job searches and job displacement for workers of any type i. This setup allows for imperfect risk sharing *across* families, that is, across any given education and ability group, but abstracts from imperfect risk sharing *within* a family, that is, within any such group. If we allowed for a lower degree of risk sharing, then the welfare losses from large minimum wage increases, or the gains from a large expansion of the EITC, would be even larger, because workers who lose their jobs would experience a larger decline in consumption, whereas workers whose employment is subsidized would enjoy a larger increase in consumption, respectively.[8]

The utility function of a family of type i is $\sum_{t=0}^{\infty} \beta^t u(c_{it}, n_{it}, s_{it})$, where c_{it} is the consumption of a representative family member, n_{it} is the index of the disutility of work of the family, and $s_{it} = \sum_j s_{ijt}$ are the total searchers, where s_{ijt} denotes the number (measure) of members searching

[7] We take the number of firms J to be large enough to be well-approximated by $J \to \infty$; see Burdett and Judd (1983). We can then interpret the sums over j in the expressions to follow as approximations to integrals over, say, a unitary measure of firms. Importantly, we assume that members of different families of any given type i are *anonymous* to firms in that firms cannot distinguish among them. This feature will imply that it is infeasible for firms to offer employment contracts that are nonlinear in the number of employed members of a particular family of any type.

[8] One way to do so is to add home production to this environment, which represents any combination of home-produced goods and transfers from the government to non-employed family members with a value smaller than the average value of the output of employed family members. Imperfect risk sharing then arises if all non-employed members of a family are *hand-to-mouth* consumers in that they simply consume their home-produced goods and all employed members share risk with each other as before. In this case, when an increase in the minimum wage increases the pool of non-employed, these workers fare worse than in our baseline model because their consumption falls more, which we conjecture will make the minimum wage less attractive. Likewise, we conjecture that an expansion of the EITC is even more attractive in this case because it reduces the pool of the non-employed.

for jobs at firm j in t. The index of the disutility of work is defined as

$$n_{it} = \left(\sum_j n_{ijt}^{\frac{1+\omega}{\omega}} \right)^{\frac{\omega}{1+\omega}} \text{ with } \omega > 0, \tag{1}$$

where n_{ijt} is the number (measure) of family members working at firm j in t. The parameter ω measures the substitutability of employment at different firms in terms of workers' disutility of work at them and can be interpreted as arising from workers' idiosyncratic preferences over different firms, locations, or amenities. The smaller ω is, the less substitutable jobs at the same firm are. Here we adapt the standard way of modeling imperfect substitutability in consumers' preferences across differentiated goods to modeling imperfect substitutability in workers' preferences across differentiated jobs. This feature leads to an upward-sloping labor supply curve for each firm's jobs analogous to the downward-sloping consumer demand curve for each firm's goods in models of monopolistic competition; see Appendix A.[9] As discussed below, ω is a key parameter governing the extent of firms' monopsony power, which we discipline in our quantitative analysis by relying on estimates of the degree to which wages are marked down relative to workers' marginal products.

Production Technology. A large number of identical firms indexed by j produce the same homogeneous final good. Firm j uses capital k_{jt}, an aggregate of efficiency units of low-educated labor $\bar{n}_{\ell jt}$, and an aggregate of efficiency units of high-educated labor $\bar{n}_{hjt}$. As noted, consumers view the labor services supplied to different firms as differentiated. The law of motion for capital is $k_{jt+1} = (1-\delta)k_{jt} + x_{jt}$, where δ is the depreciation rate and x_{jt} is the investment in new capital in period t. The same nested CES production function underlies both the version of our model with standard (putty-putty) capital and the long-run properties of the model with putty-clay capital,

$$F(k_{jt}, \bar{n}_{\ell jt}, \bar{n}_{hjt}) = \left[\psi (k_{jt})^{\frac{\alpha-1}{\alpha}} + (1-\psi) G(\cdot)^{\frac{\alpha-1}{\alpha}} \right]^{\frac{\alpha}{\alpha-1}}, \; G(\cdot) = \left[\lambda \left(\bar{n}_{\ell jt}\right)^{\frac{\rho-1}{\rho}} + (1-\lambda) (\bar{n}_{hjt})^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}. \tag{2}$$

The outer nest in $F(\cdot)$ is a CES production function over capital k_{jt} and an aggregate $G(\bar{n}_{\ell jt}, \bar{n}_{hjt})$ of the efficiency units of low- and high-educated labor, $\bar{n}_{\ell jt}$ and $\bar{n}_{hjt}$. The inner nest in $G(\cdot)$ is a CES production function over $\bar{n}_{\ell jt}$ and $\bar{n}_{hjt}$. The parameters α and ρ are important as they capture the degree of substitutability among inputs: the larger α is, the more substitutable capital and the labor aggregate $G(\bar{n}_{\ell jt}, \bar{n}_{hjt})$ are, whereas the larger ρ is, the more substitutable the low- and high-educated labor aggregates are. The parameter ρ governs the strength of firms' incentive to

[9] See Berger, Herkenhoff and Mongey (2022a) and Deb, Eeckhout and Warren (2021) for related preferences and a discussion of various interpretations and alternative microfoundations of them. This specification can be primitively derived from idiosyncratic shocks to the value of working at any firm due, say, to jobs' different locations or amenities.

substitute *away* from low-educated workers towards high-educated ones, or *towards* them, when a policy change makes low-educated workers, respectively, more or less expensive to employ. The parameter α, instead, governs how much firms substitute from the labor aggregate as a whole towards capital, or away from it, when the overall cost of labor changes. We show below that our results are largely insensitive to different estimates of capital-labor substitutability from the literature.

As for the inner nest, the labor inputs $\bar{n}_{\ell jt}$ and $\bar{n}_{hjt}$ used by firm j are themselves CES aggregates of the labor inputs of workers of different abilities within each education group,

$$\bar{n}_{\ell jt} = \left[\sum_{i \in I_\ell} z_i (\mu_i n_{ijt})^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \text{ and } \bar{n}_{hjt} = \left[\sum_{i \in I_h} z_i (\mu_i n_{ijt})^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}, \tag{3}$$

where n_{ijt} is the amount of labor of a family of type i supplied to firm j in t and $\mu_i n_{ijt}$ is the total amount of labor from all families of type i supplied to firm j in t. The parameter ϕ will play a critical role in our assessment of the distributional effects of policies, because it governs the extent to which firms are willing to substitute *across* workers of differing ability, as indexed by z, *within* an education group. For instance, if low-ability workers of a given education become more expensive due to an increase in the minimum wage, firms can substitute towards higher-ability workers with the same education—quantitatively, this margin of substitution will turn out to be the most important one. Our production structure as a function of labor in (2) and (3) is specified as in Card and Lemieux (2001). Although we augment their production structure with capital, we recover values of ϕ very close to those in Card and Lemieux (2001), which estimates a relatively high elasticity of substitution between workers of different ability within an education group.

Putty-Clay Capital. Building on Johansen (1959) and Calvo (1976), we specify the output technology as CES ex ante but Leontief ex post: once a machine is built, it is clay-like in that it requires a fixed amount of labor of *each* type to operate at full capacity. Hence, given a stock of machines, the demand for any type of workers is inelastic in the short run as long as total profits from operating the installed machines are positive, because a firm cannot substitute between existing capital and any type i of labor. Over time, though, new machines embodying new labor-to-capital ratios can be installed and so firms can substitute away from types of labor that become more expensive. For instance, when the minimum wage increases, firms can substitute away from lower-ability low-educated workers towards higher-ability workers, high-educated workers, and capital.

Formally, let $\tilde{F}(\cdot)$ denote $F(k_{jt}, \bar{n}_{\ell jt}, \bar{n}_{hjt})$ after substituting for $\bar{n}_{\ell jt}$ and $\bar{n}_{hjt}$ from (3). Omitting the subscripts j and t for simplicity, note that any constant-returns-to-scale production function such as $\tilde{F}(\cdot)$ in the inputs $(k, n_{\ell 1}, \ldots, n_{\ell M}, n_{h1}, \ldots, n_{hM})$ can be expressed in intensive form as

$$\tilde{F}(k, n_{\ell 1}, \ldots, n_{\ell M}, n_{h1}, \ldots, n_{hM}) = k\tilde{F}\big(1, \underbrace{n_{\ell 1}/k}_{v_{\ell 1}}, \ldots, \underbrace{n_{\ell M}/k}_{v_{\ell M}}, \underbrace{n_{h1}/k}_{v_{h1}}, \ldots, \underbrace{n_{hM}/k}_{v_{hM}}\big) = kf(v),$$

where $v = (v_{\ell 1}, \ldots, v_{\ell M}, v_{h1}, \ldots, v_{hM})$ denotes the *ability intensity* or *type* of capital k, that is, how much labor of each education and ability that amount of capital of type v needs in order to produce a certain amount of output. With putty-clay capital, the ratios of high- and low-educated labor of each ability to k are *fixed* at v once that capital is installed so that k units of capital of type v combined with the vector $(n_{\ell 1}, \ldots, n_{\ell M}, n_{h1}, \ldots, n_{hM})$ of labor inputs produce output

$$y(v) = \min\{k(v), n_{\ell 1}/v_{\ell 1}, \ldots, n_{\ell M}/v_{\ell M}, n_{h1}/v_{h1}, \ldots, n_{hM}/v_{hM}\}f(v),$$

according to a Leontief technology.[10] Since fixing the ratios of each type of labor to $k(v)$ amounts to fixing *all* labor ratios $n_{gm}/n_{g'm'}$ as well, this setting differs from the standard case of capital adjustment costs, since here capital as well as all other inputs are fixed in the short run.[11]

Each v_i specifies the type-i labor-to-capital ratio necessary to operate a machine of type v at full capacity. As a result, $k(v)$ units of capital of type v provide $k(v)$ units of capital services only if this capital is combined with at least $n_i = v_i k(v)$ units of labor of each type i. If $n_i > v_i k(v)$, then the excess workers remain idle, whereas if $n_i < v_i k(v)$, then the excess capital remains idle. If $k(v)$ units of capital are combined with $n_i = v_i k(v)$ units of labor of each type i, then $k(v)f(v)$ units of output are produced. In general, $\min\{k(v), \{n_i(v)/v_i\}\}/k(v) \in [0, 1]$ can be thought of as the utilization rate of $k(v)$ units of capital of type v. As input prices change, firms find it optimal to invest in new types v of capital. The total output of a firm in period t is thus

$$y_t = \int_v \min\left\{k_t(v), \{n_{it}(v)/v_i\}\right\} f(v)dv.$$

Firms choose how much to invest $x_t(v)$ in each type-v capital with accumulation law

$$k_{t+1}(v) = (1 - \delta)k_t(v) + x_t(v). \tag{4}$$

Importantly, each type v of capital is subject to the nonnegativity constraint $x_t(v) \geq 0$. Without

[10] We can nest standard capital, putty-clay capital, and intermediate versions of these two cases by allowing investment $x_t(v)$ in each type of capital v to be partially reversible via the law of motion of capital $k_{t+1}(v) = (1-\delta)k_t(v) + x_t(v)[1 + \gamma(x_t(v)/k_t(v))]$, where $\gamma(\cdot) = 0$ if $x_t(v) \geq 0$ and is a smooth positive function otherwise. So, converting $x_t(v)$ units of existing capital back to the final output good uses $x_t(v)[1 + \gamma(x_t(v)/k_t(v))]$ units of capital.

[11] For example, if production is weakly separable in capital and labor, as captured by the production function $F(k_{jt}, g(\bar{n}_{\ell jt}, \bar{n}_{hjt}))$ with $\bar{n}_{\ell jt}$ and $\bar{n}_{hjt}$ as above, then the ratio of the marginal products of any two labor inputs i and i' do not depend on capital. Hence, labor inputs *instantly* adjust regardless of the adjustment costs on capital, including when these adjustments costs are infinite.

such constraints, this technology is equivalent to the standard putty-putty technology: in each period firms could convert existing capital back into consumption goods and then back into another type of putty-clay capital and reproduce identical outcomes as under a putty-putty technology.

Matching Technology. We pin down labor market outcomes in a tractable and internally consistent way in the presence of potential labor market rationing, when policy-mandated constraints, such as the minimum wage, bind, by considering a directed search setting for labor markets. First, such a framework allows us to incorporate the non-participation margin of workers' labor supply decisions, to capture the idea that the minimum wage and the EITC incentivize non-participants to enter the labor market and search for jobs. Second, it allows us to generalize the notion of a firm-specific labor supply curve in Robinson (1933) to a framework with dynamic wage contracting, as described below, multi-worker firms, and endogenous job creation.[12] Formally, we embed the production structure described so far within a monopsonistically competitive matching framework with directed search. As in standard competitive search, each firm j posts a measure of vacancies $\mu_i a_{ijt}$ directed at consumers of type i, where a_{ijt} denotes the measure of vacancies posted by firm j per family of type i and s_{ijt} denotes the measure of searchers for jobs at firm j per family of type i in t. The cost of posting a measure $\mu_i a_{ijt}$ of vacancies for type-i consumers is $\kappa_i \mu_i a_{ijt}$.[13] The matches created by a measure $\mu_i a_{ijt}$ of vacancies and a measure $\mu_i s_{ijt}$ of searchers of type i in period t are determined by the Cobb-Douglas matching function $m(\mu_i a_{ijt}, \mu_i s_{ijt}) = B_i(\mu_i a_{ijt})^{\eta}(\mu_i s_{ijt})^{1-\eta}$.

If firm j posts $\mu_i a_{ijt}$ vacancies for type-i consumers and, in total, $\mu_i s_{ijt}$ consumers of type i search for that firm's jobs, then firm j creates a measure $m(\mu_i a_{ijt}, \mu_i s_{ijt})$ of new matches with consumers of type i, where $\lambda_f(\theta_{ijt}) = m(\mu_i a_{ijt}, \mu_i s_{ijt})/\mu_i a_{ijt}$ is the probability that a posted vacancy is filled or the *job-filling rate* and $\theta_{ijt} = a_{jit}/s_{ijt}$ denotes market tightness. We can also express these new matches as $m(\mu_i a_{ijt}, \mu_i s_{ijt}) = \lambda_w(\theta_{ijt})\mu_i s_{ijt}$, where $\lambda_w(\theta_{ijt}) = m(\mu_i a_{ijt}, \mu_i s_{ijt})/\mu_i s_{ijt}$ is the probability that a consumer of type i finds a job at firm j or the *job-finding rate*.

Timing. The timing of events in a period is as follows. At the beginning of each period t, each firm j posts vacancies $\{\mu_i a_{ijt}\}$ aimed at consumers of type i, which, together with the searchers $\{\mu_i s_{ijt}\}$, determine the tightness $\{\theta_{ijt}\}$ of the markets for consumers of type i, and commits to a present value of wages $\{W_{ijt+1}\}$ for each consumer of type i who is hired in t and begins to work in $t+1$. After observing all firms' offers, each family chooses the total number of its members $\{s_{it}\}$

[12] By allowing for long-term contracts between firms and workers, this setup also adds an additional source of sluggishness in the adjustment of inputs to any changes in policy.

[13] As we discuss below, since job creation is costly, it results is an additional source of wage markdowns in the model. However, as we show, this component of the wage markdown is quantitatively small in our baseline parameterization.

searching for jobs as well as their allocation among the J firms. Such a plan specifies the number of consumers s_{ijt} who search for each firm j when confronted with the offers $\{\theta_{ijt}, W_{ijt+1}\}$, where $s_{it} = \sum_j s_{ijt}$. At the end of t, a proportion σ of matches exogenously terminate.

2.2 Family Problem

Consumers of each family i face the risk of not finding a job when looking for one and of losing a job when employed. But since there are no aggregate shocks and families consist of a large number of members, there is no family-level uncertainty either. Thus, our economy is a deterministic one in terms of aggregates. Accordingly, the date-0 budget constraint of family i is

$$\sum_{t=0}^{\infty} Q_{0,t} c_{it} \leq \sum_{t=0}^{\infty} Q_{0,t} \sum_j W_{ijt} \lambda_w(\theta_{ijt-1}) s_{ijt-1} + \psi_i \Pi_0, \tag{5}$$

where $Q_{0,t}$ is the price of the homogeneous good in period t in its units in period 0; W_{ijt} is the present value of wages of newly employed workers; Π_0 is the present value of firms' profits; and ψ_i is the share of firms owned by a family of type i. As for the first term on the right side of (5), note that if s_{ijt-1} consumers of a family of type i search for jobs at firm j in period $t-1$, then $\lambda_w(\theta_{ijt-1}) s_{ijt-1}$ of them find a job, start working in period t, and earn the present value of wages W_{ijt} in units of period-t goods. Since a consumer of type i employed at firm j in t separates from it in $t+1$ with probability σ, the transition law for consumers of type i employed at firm j in t is

$$n_{ijt+1} = (1-\sigma) n_{ijt} + \lambda_w(\theta_{ijt}) s_{ijt} \text{ for all } j, \tag{6}$$

where $\lambda_w(\theta_{ijt})$ is the job-finding rate at firm j in t for type-i consumers.

In period 0, a family of type i chooses consumption c_{it}, the number $\{n_{ijt+1}\}$ of its employed members, and the number $\{s_{ijt}\}$ of its members searching for jobs subject to the budget constraint (5), the transition law for the employment of its members at each firm (6), a nonnegativity constraint on the number of searchers at each firm $s_{ijt} \geq 0$, with $s_{it} = \sum_j s_{ijt}$ and n_{it} satisfying (1) for all t, to maximize the present value of its utility. The first-order conditions for this problem with respect to consumption, the number of employed, and the number of searchers imply that

$$-\frac{u_{ist}}{u_{ict}} = Q_{t,t+1} \lambda_w(\theta_{ijt})(W_{ijt+1} + V_{ijt+1}) \tag{7}$$

for any firm j for which consumers are actively searching, with V_{ijt+1} recursively defined as

$$V_{ijt+1} = \frac{u_{nit+1}}{u_{cit+1}} \left(\frac{n_{ijt+1}}{n_{it+1}} \right)^{\frac{1}{\omega}} + Q_{t+1,t+2}(1-\sigma) V_{ijt+2}. \tag{8}$$

Here V_{jt+1} is the discounted marginal disutility resulting from a marginal increase in the number of family i's members who work at firm j in $t+1$, of whom $(1-\sigma)$, $(1-\sigma)^2$, and so on are still employed in $t+2$, $t+3$, and subsequent periods, expressed in units of period-$t+1$ consumption.

To understand the left side of (7), note that a marginal increase in the number of consumers who search for jobs at firm j in t leads to a corresponding increase in the disutility of search by u_{sit}/u_{cit} in consumption units. The benefit of incurring this cost is that with probability $\lambda_w(\theta_{ijt})$ such consumers find jobs in period $t+1$ at firm j and receive the present value of wages W_{ijt+1} in units of period-$t+1$ consumption, net of the present value of the disutility of work V_{ijt+1}. Expressed in period-t consumption units, this expected net benefit is $Q_{t,t+1}\lambda_w(\theta_{ijt})(W_{ijt+1}+V_{ijt+1})$, which is the right side of (7). For consumers who search for jobs at firm j in t, the value of doing so must be at least as high as the value of searching for jobs at any other firm j' so that $\lambda_w(\theta_{ijt})(W_{ijt+1}+V_{ijt+1}) \geq \mathcal{W}_t \equiv \max_{j'}\{\lambda_w(\theta_{ij't})(W_{ij't+1}+V_{ij't+1})\}$ by (7). In a symmetric equilibrium in which all other firms offer the common value $\lambda_w(\theta_{it})(W_{it+1}+V_{it+1})$, this inequality reduces to

$$\mathcal{W}_t(\theta_{ijt}, W_{ijt+1}) \equiv \lambda_w(\theta_{ijt})(W_{ijt+1}+V_{ijt+1}) \geq \mathcal{W}_t = \mathcal{W}_t(\theta_{it}, W_{it+1}) \equiv \lambda_w(\theta_{it})\left(W_{it+1}+V_{it+1}\right). \quad (9)$$

We refer to this constraint as the *participation constraint*, whose right side describes consumers' value in the *common market*.[14] When firm j makes wage offers to consumers of type i, it understands it will attract them only if this constraint is satisfied. Monopsony power emerges here as firms anticipate the increasing cost of hiring more workers of any type i, as captured by the derivatives of V_{ijt+1} with respect to vacancies a_{ijt} and market tightness θ_{ijt}, which naturally leads to an upward-sloping supply curve of workers for each firm j's jobs. Although, as shown in the online appendix, the supply curve of workers for a firm in period t is a dynamic object that depends on wages and market tightness in t as well as expectations about these variables in all future periods, we can provide some simple intuition about it in steady state, assuming that preferences are of the form $U(c_i - v(n_i) - h(s_i))$ in the spirit of Greenwood, Hercowitz and Huffman (1988) as in our quantitative analysis. In this case, the participation constraint reduces to

$$\frac{\lambda_w(\theta_{ij})}{r+\sigma}\left[w_{ij} - v'(n_i)\left(\frac{n_{ij}}{n_i}\right)^{\frac{1}{\omega}}\right] = \mathcal{W}_i$$

[14] Since we focus on a symmetric equilibrium across firms, when we set up the problem of any given firm j deciding on the offer $\{\theta_{ijt}, W_{ijt+1}\}$ for a consumer of type i in period t, that firm takes as given the symmetric offers $\{\theta_{it}, W_{it+1}\}$ of all other firms in this common market as well as the measure of consumers of type i searching in this market.

in a steady state. Differentiating it with respect to w_{ij} and n_{ij} holding fixed θ_{ij}, we obtain that

$$\frac{dw_{ij}}{dn_{ij}} = \frac{v'(n_i)}{\omega n_i}\left(\frac{n_{ij}}{n_i}\right)^{\frac{1}{\omega}-1} > 0. \tag{10}$$

Likewise, differentiating the constraint with respect to $\lambda_w(\theta_{ij})$ and n_{ij} holding fixed w_{ij} gives

$$\frac{d\lambda_w(\theta_{ij})}{dn_{ij}} = \frac{\lambda_w(\theta_{ij})}{\omega}\left[\frac{\frac{v'(n_i)}{n_i}(\frac{n_{ij}}{n_i})^{\frac{1}{\omega}-1}}{w_{ij} - v'(n_i)(\frac{n_{ij}}{n_i})^{\frac{1}{\omega}}}\right] = \frac{\lambda_w^2(\theta_{ij})}{(r+\sigma)\mathcal{W}_i}\frac{dw_{ij}}{dn_{ij}} > 0. \tag{11}$$

As (10) and (11) show, the (inverse) labor supply curves with respect to wages and job-finding rates for firm j slope upward in n_{ij}. But as ω becomes arbitrarily large, the slopes of both curves converge to zero. Intuitively, firms realize that to attract more workers of any type i, they need to compensate them for their increased disutility of work either through higher wages or through a higher job-finding probability. As a result, firms end up paying lower wages and offering fewer vacancies in equilibrium, which in turn discourages labor market search. The upward-sloping labor supply curve in wages extends the notion of static upward-sloping labor supply curve first discussed by Robinson (1933) to a dynamic setting of frictional labor markets. The upward-sloping labor supply curve in job-finding rates is a novel feature of our setup. In later sections, we discuss the extent to which the policies we consider alleviate or compound both of these monopsony distortions.

2.3 Firm Problem

We focus on the firm problem with putty-clay capital, a special case of which is that with standard (putty-putty) capital. Given an initial vector of capital $\{k_0(v)\}$, firm j chooses sequences of market tightnesses $\{\theta_{ijt}\}$, vacancies $\{\mu_i a_{ijt}\}$, employed workers $\{\mu_i n_{ijt+1}(v)\}$ for each type of capital v, present value of wages $\{W_{ijt+1}\}$, and investment $\{x_{jt}(v)\}$ in each type of capital v to maximize

$$\textstyle\sum_{t=0}^{\infty} Q_{0,t}\left\{\int_v [F(k_{jt}(v),\{n_{ijt}(v)\}) - x_{jt}(v)]dv - \sum_i \left[W_{ijt}\lambda_f(\theta_{ijt-1})a_{ijt-1} + \kappa_i a_{ijt}\right]\mu_i\right\}, \tag{12}$$

subject to the transition laws for employed workers (6), the participation constraints for employed workers in (9), along with the transition law for each type of capital (4), the adding-up constraints $n_{ijt} \leq \int_v n_{ijt}(v)dv$ for employed workers of each type i across all capital types, the Leontief constraints on employed workers of each type i allocated to each type v of capital $n_{ijt}(v) \leq v_i k_{jt}(v)$, and the nonnegativity constraints $x_{jt}(v) \geq 0$ on each type of investment v each period.

To understand the constraints $n_{ijt}(v) \leq v_i k_{jt}(v)$, recall that if a firm uses $n_{ijt}(v)$ units of type-i labor such that $n_{ijt}(v) > v_i k_{jt}(v)$, then the excess labor $n_{ijt}(v) - v_i k_{jt}(v)$ is wasted, so this is never optimal. Hence, we can impose the constraints $n_{ijt}(v) \leq v_i k_{jt}(v)$ and $n_{ijt} \leq \int n_{ijt}(v)dv$ for labor of type i and accordingly dispense with the Leontief form for the production function in the firm's problem. By the non-negativity constraint $x_{jt}(v) \geq 0$, firms cannot disassemble their existing types of capital. Without this friction, the firm's problem reduces to the standard putty-putty problem. Since there are no aggregate shocks and families can perfectly insure the idiosyncratic risk of their members, it is without loss to adopt the convention that a firm fulfills its present-value wage offer W_{ijt} by offering a constant period wage w_{ijt} over the course of a match that begins at t so that

$$W_{ijt} = w_{ijt} + (1-\sigma)Q_{t,t+1}w_{ijt} + (1-\sigma)^2 Q_{t,t+2}w_{ijt} + \ldots = d_t w_{ijt}, \tag{13}$$

where $Q_{t,s}$ is the price of goods in $s > t$ in units of goods in t and $d_t \equiv 1 + (1-\sigma)Q_{t,t+1} + (1-\sigma)^2$-$Q_{t,t+2} + \ldots$. So, firms can be equivalently thought of as choosing W_{ijt} or w_{ijt}.

A (symmetric) *monopsonistically competitive search equilibrium* with putty-clay capital is a collection of allocations of consumption, employment, job searchers, labor market tightnesses, capital, and prices $\{W_{it+1}, Q_{0t}\}$ such that at these allocations and prices *i*) families' decisions are optimal for each type i, *ii*) firms' decisions are optimal, and *iii*) markets clear in each t.

Although we have not explicitly modeled firm entry, there is a simple way of allowing for it that ensures that our quantitative results are unaffected. To elaborate, imagine, in the spirit of Hopenhayn (1992), that there exists a large number of potential entrants that must pay a fixed cost to enter the market. Entry occurs until the cost of entry for the marginal firm is exactly balanced by the expected present discounted value of profits after entry. Firms that have previously entered, and hence now treat their entry cost as sunk, will exit at some date t only if the present value of profits from then on is negative. In our quantitative exercises, we assume that the economy begins in a steady state, say date 0, with a fixed number of firms, say J. For all of our minimum wage experiments, we explicitly check if the present value of profits from any $t > 0$ on are positive. Since they are, then no firm will exit. Clearly, no firms will enter because the present value of profits is now lower than it was in the steady state. Now consider our EITC experiments, in which we fund the subsidies to workers with a flow tax on profits. To put the EITC on the same footing as the minimum wage, we choose such a tax to exactly equal the loss in profits resulting from the specific minimum wage experiment we compare the EITC to. Hence, when we check if firms want to exit at any $t > 0$ after the imposition of the EITC, we also find that they do not want to. Likewise, no firms will enter because the tax on profits offsets the subsidies to workers who receive one under the EITC. In sum, our model is consistent with endogenous firm entry.

2.4 Equilibrium Characterization

The novel part of our model is the firm's problem, which is hard to characterize. One difficulty is that we capture monopsonistically competitive behavior by making firms face a forward-looking participation constraint for each type of worker hired in each period t, as described. To deal with these constraints, we follow the approach in Marcet and Marimon (2019) and aggregate the infinite sequence of Lagrange multipliers on them into one variable for each worker type that recursively evolves over time. A second difficulty is that, in general, we need to record a possibly infinite vector of capital types in each period, which would be numerically infeasible for our later policy exercises. To circumvent this difficulty, we proceed as follows. We posit a restricted problem in which a firm must fully utilize all types of capital it holds. We then show that we can reduce this restricted problem to an equivalent tractable one, in which the only aggregate state variables, which evolve recursively, are aggregate output and the aggregate employment of each consumer type. We prove that for this equivalent problem, in each period t along the transition to a new steady state, firms invest in only one type of capital, $\{v_t\}$. Finally, we rely on the solution to this equivalent problem to construct a candidate solution to the original problem and provide simple necessary and sufficient conditions for this candidate solution to solve the original problem, which we verify are satisfied.

2.5 Steady-State Properties

Note that the steady states of the model with standard capital and of that with putty-clay capital are identical. Intuitively, once factor prices and the intertemporal prices of consumption that firms face become constant, firms with putty-clay capital invest in the only type of capital that is best suited to their technologies at those prices and let all past capital depreciate. Eventually, all the old capital stock is replaced. In particular, in a steady state with putty-clay capital, firms invest in exactly the same type of capital as they do in a steady state with standard capital. We focus on preferences in the spirit of Greenwood, Hercowitz and Huffman (1988), $u(c_i, s_i, n_i) = U(c_i - v(n_i) - h(s_i))$. Consider the steady state of our model, in which all variables are constant. A firm's Euler equation for capital is $r = \tilde{F}_k + \delta$ with $1 + r = 1/\beta$, a firm's vacancy posting condition is

$$\frac{\kappa_i}{\lambda_f(\theta_i)} = \frac{1}{r+\sigma}\Big[\tilde{F}_{ni} - w_i - \frac{v'(n_i)}{\omega}\Big], \tag{14}$$

the first-order condition for the number of searchers for a family of type i is

$$h'(s_i) = \frac{\lambda_w(\theta_i)}{r+\sigma}[w_i - v'(n_i)], \tag{15}$$

and equilibrium wages satisfy

$$w_i = \eta \left[\tilde{F}_{ni} - \frac{v'(n_i)}{\omega} \right] + (1-\eta) v'(n_i), \tag{16}$$

where $\tilde{F}_k = \tilde{F}_k(k, \bar{n}_\ell(n_i), \bar{n}_h(n_i))$, $\tilde{F}_{ni} = \tilde{F}_{ni}(k, \bar{n}_\ell(n_i), \bar{n}_h(n_i))$ and $\bar{n}_\ell$ and $\bar{n}_h$ satisfy the symmetric steady-state version of (3). The steady-state law of motion for employment reduces to

$$\lambda_w(\theta_i) s_i = \sigma n_i. \tag{17}$$

Consumption satisfies a steady-state version of (5). Since our preferences imply no income effects, the steady-state conditions split into two blocks. First, we can solve for the monopsonistically competitive search equilibrium wages $\{w_i\}$ and the associated allocations $\{n_i, s_i, \theta_i\}$ and k from (14)–(17). Then, given these allocations and prices, we can solve for consumption $\{c_i\}$ from (5).

Firms' monopsony power distorts both wages and vacancy posting by the term $v'(n_i)/\omega$. For a given marginal product of labor and marginal disutility of work, a firm offers a smaller wage and posts fewer vacancies than it would under the competitive search equilibrium for our economy—the efficient case—which corresponds to the case in which $v'(n_i)/\omega = 0$ as $\omega \to \infty$. Since vacancies are distorted downward, so is the probability of finding a job $\lambda_w(\theta_i)$. As apparent from (15), inefficiently low wages and job-finding probabilities depress the marginal benefit of labor market search and so result in consumers searching too little for jobs. Firms' vacancy-posting condition (14) features both the indirect distortion from an inefficient level of wages and the direct distortion from the term $v'(n_i)/\omega$, due to the increasing marginal cost of hiring additional workers of any given type.

Since a firm with monopsony power pays its workers only a fraction of their marginal products, a simple measure of firms' monopsony power is then the markdown of wages relative to their marginal products, $1 - w_i/\tilde{F}_{ni}$. In a slight abuse of language, we refer to $w_i/\tilde{F}_{ni}$ itself as the *wage markdown*. Letting $\kappa(\theta_i) = (r+\sigma)\kappa_i/\lambda_f(\theta_i)$, we can combine the equilibrium vacancy-posting condition and the wage equation to show that the implied wage markdown for workers of a family of type i is

$$\frac{w_i}{\tilde{F}_{ni}} = \left[1 + \underbrace{\frac{\kappa(\theta_i)}{\frac{\eta}{1-\eta}\kappa(\theta_i) + v'(n_i)}}_{\text{efficient component}} + \underbrace{\frac{\frac{1}{\omega} v'(n_i)}{\frac{\eta}{1-\eta}\kappa(\theta_i) + v'(n_i)}}_{\text{monopsony component}} \right]^{-1}. \tag{18}$$

In (18), the *efficient component* of the markdown is defined as the markdown that would arise in the absence of firms' monopsony power, and corresponds to the level of the wage markdown needed for firms to recoup their vacancy-posting costs and hence earn zero expected profits per vacancy.

More interesting is the *monopsony component*, which arises because firms with monopsony power set wages below the competitive search level and leads to larger markdowns than under competitive search. As this equation makes clear, for any given size of measured markdowns, the larger the efficient component is, the smaller the monopsony component is. In our quantitative exercises, we find that the overwhelming majority of the markdown in wages is due to the monopsony component.

Minimum Wage Implications. As discussed, as ω becomes arbitrarily large and jobs then become arbitrarily substitutable in workers' preferences, the monopsonistically competitive search equilibrium wage and vacancy-posting conditions converge to their competitive search equilibrium counterparts. In this case, wages, job creation, and search converge to their efficient levels.

***Lemma* 1.** *As $\omega \to \infty$, wages and allocations in the monopsonistically competitive search equilibrium converge to those of the competitive search equilibrium.*

In our framework, a policy that mandates that firms pay a *type-specific* minimum wage $\underline{w}_i$ for each consumer type i, set equal to the wage for that worker type in the competitive search equilibrium, would correct the distortions to wages but not to vacancies. To correct that latter, a hiring mandate would also be needed. We summarize this discussion in the following result.

***Proposition* 2.** *If the minimum wage for each worker type is set equal to the competitive search equilibrium wage for that type, hiring is inefficiently low. If a type-specific minimum wage is coupled with a mandate to hire as many workers of each type as implied by the competitive search equilibrium, then the associated wages and allocations coincide with those of the competitive search equilibrium.*

We have illustrated the ideas behind these results for a steady state, but their analogs hold for a dynamic equilibrium. The issue we address in later sections is that although a rich set of type-specific minimum wages could in principle fix the *wage component* of the distortions due to firms' monopsony power—which turns out to be quantitatively much more important than the job-creation component of these distortions—in practice setting such a complex minimum-wage system is infeasible. (In Section 7, we discuss the sense in which European systems that feature multiple minimum wages approximate these policies.) We then focus on the other extreme that corresponds to the federal minimum wage policy in the United States, which consists of *one* mandated minimum for all workers. Intuitively, if large enough differences in education and ability exist across workers, then a single minimum wage that is too high can have perverse distributional effects.

Tax and Transfer Program Implications. Consider a general tax and transfer system in which $T(w_i)$ denotes labor income taxes and $-T(w_i)$ denotes labor income subsidies. Let $A(w_i) =$

$w_i - T(w_i)$ denote after-tax labor income. Such a system affects both the incentives for firms to hire workers and for households to search for jobs. Consider first how the system influences firms' labor demand, as summarized by the steady-state vacancy posting condition

$$\frac{\kappa_i}{\lambda_f(\theta_i)} = \frac{1}{r+\sigma}\left[\tilde{F}_{ni} - w_i - \frac{v'(n_i)}{\omega A'(w_i)}\right], \tag{19}$$

where $A'(w_i)$ is the marginal after-tax income. Equation (19) shows that a positive marginal tax rate, which implies $A'(w_i) < 1$, exacerbates monopsony distortions, relative to our baseline model with $A'(w_i) = 1$, by increasing the magnitude of the last term in (19) in absolute value and so firms' cost of hiring. To understand this mechanism, recall that a monopsony distortion arises because hiring a marginal worker increases the marginal disutility of work for all inframarginal workers of the same type. So, a firm needs to compensate inframarginal workers with a higher wage, which increases its cost of hiring. A positive marginal tax rate reduces the after-tax wage that inframaginal workers receive and therefore increases the required before-tax wage that a firm must offer, further increasing the cost of hiring. Conversely, a negative marginal tax rate, namely, a tax credit resulting in $A'(w_i) > 1$, alleviates monopsony distortions by reducing firms' costs of hiring—that is, it reduces the last term on the right side of (19).

Whereas *marginal* tax rates affect monopsony distortions to firms' labor demand, *average* tax rates influence households' labor supply by affecting their search decisions. Specifically, under a tax and transfer system, the optimal search decision for a family of type i satisfies

$$h'(s_i) = \frac{\lambda_w(\theta_i)}{r+\sigma}\left[A(w_i) - v'(n_i)\right]. \tag{20}$$

A positive average tax rate, which implies $A(w_i) < w_i$, reduces the after-tax wage relative to our baseline model, thereby depressing the incentive to search . By contrast, a negative average tax rate implies $A(w_i) > w_i$ and so stimulates search. This discussion suggests that policies part of the existing tax and transfer system, such as the EITC, that more directly target average and marginal wages may be more effective at alleviating monopsony distortions than the minimum wage.

3 Quantification

We choose the parameters of our model to reproduce important features of U.S. labor markets, which inform its mechanisms. To start, we assume that a model period is one month in order to adequately capture worker transitions into and out of jobs. In the spirit of Greenwood, Hercowitz and Huffman (1988), we maintain that the utility function of a household has the form

$$u(c_{it}, n_{it}, s_{it}) = \log\left(c_{it} - \chi_{gn}\frac{n_{it}^{1+1/\gamma_n}}{1+1/\gamma_n} - \chi_{gs}\frac{s_{it}^{1+1/\gamma_s}}{1+1/\gamma_s}\right), \tag{21}$$

where χ_{gn} governs the disutility of work for education group $g \in \{\ell, h\}$, χ_{gs} governs the disutility of search for each such group, and γ_n and γ_s control the elasticity of labor supply and job search.

3.1 Disciplining Key Features of the Model

We start by summarizing how we parameterize key aspects of our model: the degree of firms' monopsony power, the distribution of worker heterogeneity, the elasticities of substitution across workers and between capital and labor, and a crucial element of the putty-clay technology, namely the depreciation rate of physical capital. Table 1 reports the key parameters of our model. Although the values of the parameters that are endogenously pinned down are jointly determined, we provide next some intuition about how they are disciplined by specific moments in the data.

TABLE 1: Key Parameters

Parameter	Description	Value	Discipline
Monopsony power			
ω	Substitutability of jobs across firms	2.85	Match estimates of wage markdowns
Worker heterogeneity			
μ_i	Distribution of productivities z_i	lognormal	Match wage distribution from ACS
Long-run elasticities of substitution			
α	Elasticity of substitution between capital and labor	1.00	Fixed: Cobb-Douglas
ρ	Elasticity of substitution between high- and low-educated workers	1.40	Match moment/estimated value in Katz and Murphy (1992)
ϕ	Elasticity of substitution across workers within an education group	4.00	Match moment/lower-bound of estimated values in Card and Lemieux (2001)
Putty-clay frictions			
δ	Depreciation rate	15% annual	Match equipment and software (BEA)

Monopsony Power. The degree of monopsony power is crucial for our policy exercises, as it determines the scope for policies such as the minimum wage and the EITC to increase employment. In our model, the degree of monopsony power is controlled by the substitutability of jobs in workers' preferences, as captured by the parameter ω. We discipline ω by targeting existing empirical estimates of the average wage markdown, which are informative because, as equation (18) shows, ω determines the size of the monopsony component of the wage markdown. As ω becomes large, firms' monopsony power decreases and the inefficient component of the wage markdown becomes small. A growing literature has measured wage markdowns in the United States and estimated that, on average, workers are paid between 0.65 and 0.85 of their marginal products; in our baseline, we target the midpoint of this range, 0.75.[15] In Section 7, we show the sensitivity of our results to higher and lower estimates of wage markdowns.

Worker Heterogeneity. The distribution of workers' productivity within an education group captured by $\{\mu_i\}$ governs the degree of wage dispersion within each such group and thus the distributional impact of policies. For each education group $g \in \{\ell, h\}$, we assume this distribution is lognormal, so z has a common support across groups and $i = (g, z)$, with group-specific standard deviation σ_g. We choose $\{\sigma_g\}$ to match the dispersion of wages for each education group from the 2017-2019 American Community Survey (ACS).[16] Specifically, we target the ratio of the 50th to the 10th percentiles of the wage distribution of each group in order to reproduce well the left tail of each distribution, which is most directly affected by the redistributive policies we examine.

Long-Run Elasticities of Substitution. The production function parameters α, ρ, and ϕ govern the long-run substitutability between capital and labor and between different types of labor. An active debate in the literature concerns the elasticity of substitution between capital and labor, here measured by α, which has been found to range from a value suggesting greater complementarity than Cobb-Douglas in Oberfield and Raval (2021), namely, $\alpha = 0.5$ to $\alpha = 0.7$, to a value suggesting greater substitutability than Cobb-Douglas in Karabarbounis and Neiman (2014), namely, $\alpha = 1.25$. As a compromise between these estimates, we set $\alpha = 1$ (Cobb-Douglas) as our baseline, but, as we discuss in Section 6, our results are essentially invariant to values of α in this range.[17]

We pin down the long-run elasticities of substitution across workers ρ and ϕ as in Card and Lemieux (2001), which estimates a long-run CES production structure similar to ours that nests workers with different levels of experience within an education group, interpreted as corresponding to different levels of ability as in our model. The within-group elasticity ϕ is particularly important for our policy analysis, because it controls the extent to which firms substitute away from low-wage workers in response to an increase in the minimum wage or towards them in response to the EITC or its expansion. Card and Lemieux (2001) report estimates of ϕ between 4 and 6.[18] In our

[15] See, for example, Seegmiller (2021), Lamadon, Mogstad and Setzler (2022), Berger, Herkenhoff and Mongey (2022a), and Yeh, Macaluso and Hershbein (2022). Manning (2021) provides a recent survey of this literature.

[16] We restrict the sample to include all individuals aged 16 and above, exclude all individuals residing in group quarters, and those who report themselves as being students, which mirrors the sample restrictions used by the BLS to compute labor market statistics. Given that we exclude students from our analysis, there are very few individuals in our sample between the ages of 16 and 21. As a result, our key findings are essentially unchanged if we calculate the wage distribution using a sample of individuals aged 21 and above. See Appendix B for additional details.

[17] In Appendix E.3, we study an alternative version of our model, in which capital and non-college labor are substitutes but capital and college labor are complements, as in Krusell et al. (2000). Results for this version are almost identical to our baseline results, since the labor-labor elasticity of substitution is much larger than the elasticity of substitution between labor and capital, which makes the labor-labor margin of substitution the dominant one.

[18] These authors use data for the United States, the United Kingdom, and Canada, and estimate elasticities between

baseline, ϕ equals 4 as in the lower end of this range; we explore the robustness of our results to higher and lower values of ϕ in Section 7. As Card and Lemieux (2001) find similar estimates for both college-educated and non-college-educated workers, we assume the same value of ϕ for both high- and low-educated workers.

Card and Lemieux (2001) estimate the between-education-group elasticity of substitution ρ by exploiting long-run variation in the wages and relative supply of aggregate labor inputs controlling for trends in labor demand, by an approach similar in spirit to the seminal work of Katz and Murphy (1992). We choose $\rho = 1.4$ to be consistent with the benchmark value in Katz and Murphy (1992), which is within the range of estimates documented by Card and Lemieux (2001) as well.[19] Taken together, these estimates imply that the elasticity of substitution among workers *between* education groups is much smaller than that *within* an education group, that is, $\rho < \phi$. Below, we discuss how our key distributional results are affected only in a minor way when we use higher estimates of the between-education-group elasticity of substitution.

In this quantification strategy, we have relied as much as possible on parameter values that have been validated by other studies, so as to base our policy exercises on characteristics of preferences and technology with broad consensus in the literature and hence guarantee that these exercises are in no way skewed by arbitrary choices of parameter values. One question arises, however, as to whether these key parameter values that inform our baseline are appropriate for our model. To address this question, we show that the elasticities ρ and ϕ estimated by Card and Lemieux (2001) closely map into the values of these parameters implied by our model, although the framework in Card and Lemieux (2001) does not feature either firm monopsony power or search frictions.

We do so in two ways; see Appendix C. First, we treat the estimated values of 1.4 and 4 for ρ and ϕ in Card and Lemieux (2001) as the parameter estimates of an auxiliary model defined by Card and Lemieux (2001)'s estimating equations, according to the indirect inference procedure described in Appendix C. This procedure yields values of ρ and ϕ that are essentially the same as those in our baseline. Second, we apply the estimation procedure in Card and Lemieux (2001) to data simulated from our model at their estimated parameter values and recover these parameter values almost exactly. The reason for why both approaches lead to values of ρ and ϕ so similar to those in our

3.8 and 4.9 for the United States using Census and Current Population Survey (CPS) data between 1959 and 1996. Similarly, Borjas (2003) estimates a within-education-group labor-labor elasticity of substitution close to 4 by exploiting changes in immigration patterns over time. Relying on a similar approach, Borjas and Katz (2007) reports a slightly lower estimate of about 3 using more recent IPUMS data.

[19] Card and Lemieux (2001) reports very similar results to those in Katz and Murphy (1992) when considering both men and women. Other work estimates higher elasticities of substitution between non-college and college workers between 2.5 and 4. See, for example, Borjas and Katz (2007) and Bils, Kaymak and Wu (2020).

baseline is that Card and Lemieux (2001)'s estimating equations are an approximate reduced form of our model. Namely, the markdown of wages relative to workers' marginal products in our model is roughly constant across workers; see Figure 1. Thus, the ratios of the wages of different types of workers, which are the key moments in the estimation procedure in Card and Lemieux (2001), approximately equal the ratios of their marginal products, as in Card and Lemieux (2001).[20]

Capital Depreciation Rate. Because of our putty-clay technology, the dynamic effects of policies depend on the rate at which firms adjust their capital stock. The speed of this adjustment process is largely determined by the depreciation rate of capital, δ. We set this rate to reproduce an annual depreciation rate of 15%, which matches the aggregate capital depreciation rate for the U.S. economy excluding structures, and later explore the impact of alternative values of δ.

3.2 Further Details of the Quantification Procedure

We now provide further details of our quantification procedure, including how we discipline the rest of the parameters, which are less important for our main results. We proceed in two steps: we first exogenously fix a subset of these parameters based on external evidence on their values and then choose the remaining ones in order to match moments from the data that are informative about them. The parameters governing the labor market search portion of the model—the cost of posting jobs, the parameters of the matching function, and those governing a household's disutility of labor market search—mainly determine the degree to which a change in employment resulting from a policy change manifests itself as a change in labor force participation rather than unemployment.

TABLE 2: Other Fixed Parameters

Parameter	Description	Value
γ_n	Labor supply elasticity	1.00
π_ℓ	Fraction of non-college households	0.69
β	Discount factor	$(1.04)^{-1/12}$
σ	Job destruction rate	2.8%
η	Elasticity of matching function w.r.t. searchers	0.50
γ_s	Search supply elasticity	5.00
χ_s	Scale of search disutility	3.8×10^6

Note: Other parameters in addition to those in Table 1 exogenously fixed. A model period is one month.

Table 2 shows the other parameters that we exogenously fix, in addition to those in Table 1. We set the parameter γ_n of the utility function, which primarily governs the elasticity of labor supply, to 1, but we discuss how our results are robust to a range of alternative values in Section 6. We fix the share of college-educated households to $1 - \pi_\ell = 31\%$ in order to match their proportion in the

[20] As our production structure is weakly separable in capital and labor, the ratios of the marginal products of different workers do not depend on capital. Hence, they fall into the class estimated by Card and Lemieux (2001).

ACS data. We choose a value for households' discount factor β of $(1.04)^{-1/12}$ so that the annualized real interest rate r equals 4%. We set the job destruction rate σ to 2.8% per month, which equals the Abowd-Zellner corrected estimate of the separation rate by Krusell et al. (2017), based on data from the Current Population Survey (CPS) and the elasticity of the matching function with respect to the measure of searchers to $\eta = 0.5$, as in Ljungqvist and Sargent (2017). Note that there exists a locus of values for the parameters γ_s and χ_s (this latter assumed to be common across the two education groups) governing the disutility of labor market search that imply approximately identical steady-state labor market outcomes but differ in the response of search effort to, say, a minimum wage increase. We choose a pair of values on this locus that imply a relatively muted response of search effort to the minimum wage, as in the data (Adams, Meer and Sloan (2022)).

Table 3: Fitted Parameters

Parameter	Description	Value
Monopsony power		
ω	Monopsony power	2.85
Worker productivity distribution $\log \mathcal{N}(0, \sigma_g)$		
σ_ℓ	Standard deviation for non-college z_i	0.68
σ_h	Standard deviation for college z_i	0.79
Production function		
ψ	Coefficient on capital k	0.30
λ	Coefficient on non-college labor n_ℓ	0.55
Search frictions		
B	Matching function productivity	0.47
Labor disutility		
$\chi_{\ell n}$	Scale of non-college labor disutility	1.64
χ_{hn}	Scale of college labor disutility	2.14

Note: Parameters chosen to match the statistics in Table 4.

Table 3 contains the values of the parameters chosen to match the moments in Table 4: the average wage markdown; the wage distribution, income share, and employment rate by education group; and the aggregate unemployment and job-finding rates—note that in our model, employment, unemployment, and participation differ across workers with different education and productivity. We include the monopsony power ω and the distribution of worker productivity z_i in Table 3 for completeness, although they have been discussed in Section 3.1, as the values of all parameters are jointly determined. Intuitively, as mentioned, the degree of monopsony power ω is primarily determined by the average wage markdown in the data, and the dispersion of worker productivity σ_ℓ and σ_h by education group is largely pinned down by the ratio of the 50th to the 10th percentiles of the wage distribution of each education group. The scale parameters of production ψ and λ in (2) govern the aggregate labor share and the share of labor income accruing to college-educated workers. The parameter B for the efficiency of the matching function determines the steady-state unemployment and job-finding rates—we target values of 5.9% and 45% as consistent with the level of unemployment and worker flows out of unemployment before the Great Recession (Shimer (2005) and Kehoe et al. (2023)). Finally, the parameters $\{\chi_{gn}\}$ of the disutility of work of each group g control the steady-state employment rates of each group and are disciplined by them.[21]

TABLE 4: Targeted Moments

Moment	Description	Data	Model
Average wage markdown			
$\mathbb{E}[w_{ni}]/\mathbb{E}[\tilde{F}_{ni}]$	Average wage markdown	0.75	0.76
Wage distributions			
$w_{\ell 50}/w_{\ell 10}$	Non-college 50th-10th ratio	2.04	1.98
w_{h50}/w_{h10}	College 50th-10th ratio	2.30	2.07
Income shares			
$\mathbb{E}[w_i n_i]/Y$	Aggregate income share	57%	57%
$\pi_h \mathbb{E}[w_{hz} n_{hz}]/\mathbb{E}[w_i n_i]$	College income share	55%	55%
Employment rates			
$\mathbb{E}_\ell[n_i]$	Non-college employment rate	47%	47%
$\mathbb{E}_h[n_i]$	College employment rate	62%	61%
Unemployment rate			
$\mathbb{E}[s_i]/(\mathbb{E}[s_i] + \mathbb{E}[n_i])$	Average unemployment rate	5.9%	5.9%
Job-finding rate			
$\mathbb{E}[\lambda_w(\theta_{ijt})]$	Average job-finding rate	45%	44%

Note: The average wage markdown is the midpoint of the range of estimated markdowns. The average income share is from Karabarbounis and Neiman (2014). The moments of the wage distribution by education group, college income share, and employment rates are calculated using ACS 2017-2019. The average unemployment and job-finding rates are calculated using data from the Bureau of Labor Statistics (BLS).

As Table 4 shows, the model reproduces the targeted moments well. Importantly, it matches the average wage markdown, and so the degree of monopsony, almost exactly. The non-college 50th–10th wage ratios, aggregate and college income shares, employment shares by education group, and unemployment and job-finding rates are nearly identical in the model and in the data.

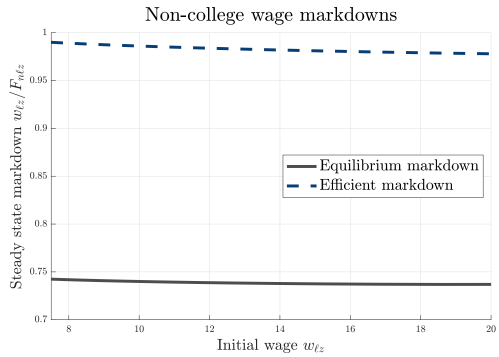
3.3 Implied Wage Distributions and Firm Monopsony Power

As we discuss in Appendix D, our model successfully reproduces the distribution of wages for workers with and without a college degree. Figure 1 plots the wage markdown implied by our model as a function of the steady-state wage $w_{\ell z}$ of non-college workers of any ability z.

Recall from expression (18) that the steady-state wage markdown consists of both an efficient and an inefficient component. The efficient component captures that firms must recoup the average annuitized cost $\kappa_i/\lambda_f(\theta_i)$ of recruiting a worker of any type i in order to break even at the time they post a vacancy. Since this component is about 1% to 2% of the average wage markdown, the bulk of it is accounted for by the remaining inefficient component, which is due to firm monopsony

[21] As in Shimer (2005), we normalize market tightness to one in steady state, which pins down the vacancy-posting cost κ_0, where $\kappa_i = \kappa_0 z_i$.

FIGURE 1: Wage Markdowns



Note: Steady-state wage markdown $w_{\ell z}/\tilde{F}_{n\ell z}$ of select z-types among non-college workers ($i = (\ell, z)$). *Equilibrium markdown* corresponds to our model. *Efficient markdown* corresponds to the equilibrium of the model without monopsony power ($\omega \to \infty$). The x-axis corresponds to the wage $w_{\ell z}$ of a type-z worker.

power and leads firms to earn substantial profits in equilibrium.

4 Short-Run and Long-Run Effects of the Minimum Wage

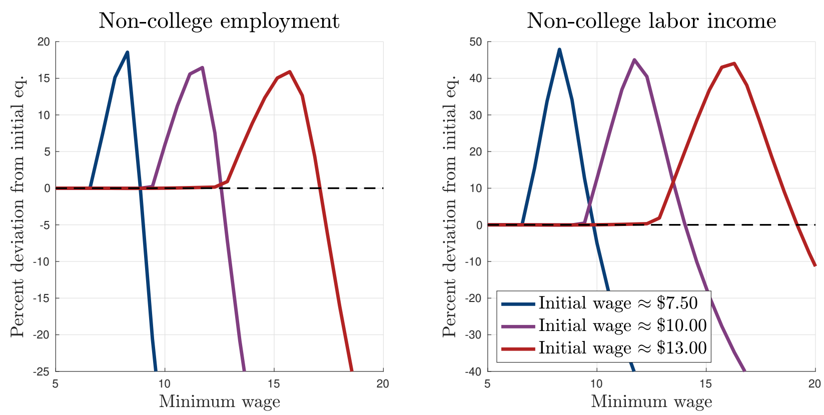
We now study the dynamic effects of increases in the minimum wage. Our main result in this section is that the short-run labor market responses to minimum wage increases differ from the corresponding long-run responses due to putty-clay adjustment frictions. We further show that the difference between short- and long-run effects is more pronounced for larger minimum wage increases. In Section 5, we will contrast these results with the effects of the EITC.

4.1 Long-Run Effects of the Minimum Wage

We begin by showing how different levels of the minimum wage affect the long-run steady state assuming that the minimum wage has been introduced in some previous period and that the economy has converged to a corresponding new steady state—in the next subsection, we examine the transition path to it. Note that for the wage distributions implied by our baseline parameterization, the current federal minimum of \$7.25 barely binds. So, we think of the initial steady state without any policy, which is our starting point, as corresponding to the current federal policy regime. The minimum wage $\underline{w}$ modifies the steady-state equilibrium conditions. For households of type i for which the minimum wage binds, the vacancy-posting condition in (14), the first-order condition for search in (15), and the steady-state wage equation in (16) are modified to be consistent with the binding minimum; see the online appendix. The remaining conditions are unchanged.

Laffer Curves for Select Workers. Figure 2 shows how different levels of the minimum wage affect individual non-college workers' employment (left panel) and labor income (right panel) in the long run.[22] For each of the three z-types of workers we consider—earning wages of \$7.50, \$10, and \$13 in the initial steady state—the effect of the minimum wage has an inverted-U, or *Laffer curve*, shape. The height and slopes of these individual Laffer curves are determined by the degree of firm monopsony power measured by ω and of labor-labor substitutability measured by ϕ.

FIGURE 2: Disaggregate Long-Run Minimum Wage Laffer Curves



Note: Percentage change in steady-state employment (left panel) and labor income (right panel) of select z-types among non-college workers as a function of the minimum wage, relative to the initial steady state.

Consider workers initially earning \$7.50 per hour (the blue lines). Given that their initial wages were inefficiently marked down due to firm monopsony power, the minimum wage increases these workers' employment until it reaches a value of around \$8.50.[23] Raising the minimum wage up to that level increases employment by reducing monopsony distortions. The larger is the degree of firm monopsony power, the higher the peak of these curves, and so the larger the employment gain from an efficient minimum wage policy. In the absence of firms' monopsony power ($\omega \to \infty$), these curves would have no such hump and would be instead strictly downward sloping. Increasing the minimum wage above the efficient level induces firms to substitute away from these workers, though. The more substitutable these workers are with other more productive workers, the steeper is the slope of the declining portion of the Laffer curves for employment, once the peak is reached. Hence, the degree of monopsony power governs the upward-sloping part of the Laffer curves, whereas the degree of labor-labor substitutability governs the steepness of the downward-sloping part.[24]

[22]We focus on non-college workers as the new minimum is barely binding for college workers. Appendix E.1 shows how college employment and labor income vary with the minimum wage at both individual and aggregate levels.

[23]In general, the peak of the Laffer curve is somewhere between a worker's initial wage and initial marginal product of labor. With our monopsony markdown of 25%, this upper bound is approximately $\$9.40 = 1.25 \times \$7.50$.

[24]The participation response (not shown) is similar to the employment response and peaks at around the same

The right panel of Figure 2 shows that the Laffer curves for labor income have a higher peak and display a slower decline when the minimum wage is raised above the efficient level, relative to the employment curves. Intuitively, a higher minimum wage directly increases the wages of the workers for whom it binds who remain employed. For example, a \$15 minimum wage doubles the income of workers initially earning \$7.50 who are still employed. However, as described, if the minimum wage is set sufficiently high, it reduces both employment and labor income for some workers.

Applying this logic across the different workers depicted in the figure reveals a long-run distributional trade-off: a single minimum wage cannot simultaneously correct monopsony distortions for all workers. The reason is simple. The minimum wage is too blunt a redistributive instrument to offset firms' monopsony power in the different markets for workers of different productivity. For example, as the minimum wage increases from its current level by \$1.25, from \$7.25 to \$8.50, the employment of low-productivity workers increases, whereas the employment of higher-productivity workers is unaffected. However, further increasing the minimum wage to, say, \$15 helps alleviate the monopsony distortions of only higher-productivity workers and at the cost of decreasing the employment of lower-productivity workers, such as those initially earning \$7.50 or \$10.00. Indeed, even focusing only on these three types of low-wage workers, no single minimum wage exists that simultaneously increases either their employment or income. This figure thus reveals the distributional paradox of too large a minimum wage policy: in the long run without any countervailing force such as inflation, productivity growth, or other corrective policy, it hurts precisely the lowest-earning workers whose income it is supposed to support.

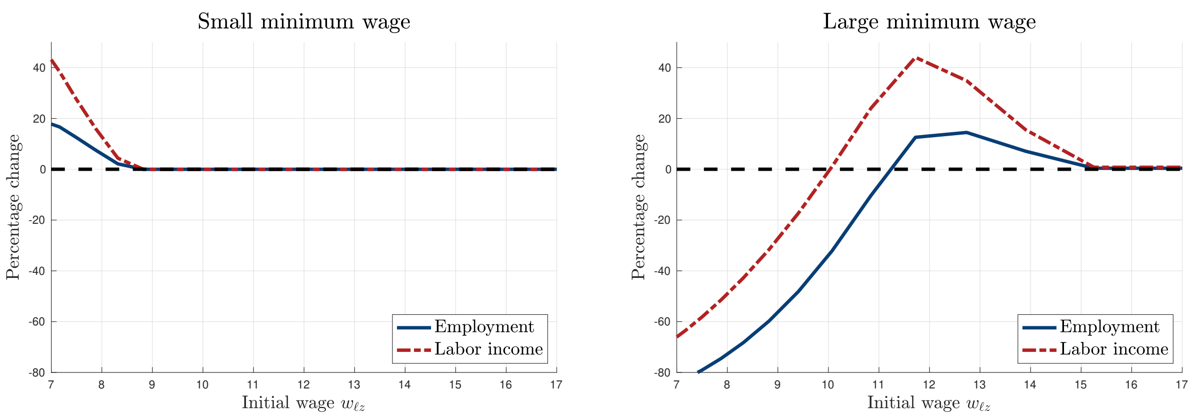
Small vs. Large Minimum Wages Across the Distribution of Workers. Figure 3 provides further insight about this long-run distributional trade-off by contrasting the effects of a small and a large increase in the minimum wage across non-college workers. To fix ideas, we set the small minimum wage at \$8.50 (a 17% increase over \$7.25), which is on the higher end of the minimum wage increases commonly explored in the empirical literature (see, for example, Cengiz et al. (2019)). We set the large minimum wage to \$15, which is the level currently discussed in the United States as a potential federal minimum. The left panel of Figure 3 shows that a small increase in the minimum wage is beneficial for the lowest-earning workers: it increases the employment and labor income of all workers initially earning less than \$8.50 and has no negative impact on the remaining workers.

By contrast, the right panel of Figure 3 shows that a large minimum wage increase has negative effects for the lowest-earning workers. Employment falls for all non-college workers initially earning

value of the minimum wage for each type.

less than \$11 per hour, who account for 26% of all non-college workers and 62% of the workers for whom the minimum wage binds. This decline is the largest among the lowest-earning workers, because the efficient levels of the minimum wage for them are substantially below \$15. When employment gains occur, they are concentrated among workers whose initial wages are already close to \$15. The same broad pattern holds in terms of labor income, although the group of workers whose labor income falls is smaller, as wages rise for all those who remain employed.

FIGURE 3: Long-Run Distributional Effects of \$8.50 and \$15 Minimum Wage



Note: Percentage change in steady-state employment and labor income for select z-types among non-college workers for an \$8.50 minimum wage (left panel) and a \$15 minimum wage (right panel), relative to the initial steady state. The x-axis corresponds to the wage $w_{\ell z}$ of a type-z worker in the initial steady state.

Taken together, these results highlight the blunt nature of the minimum wage as a redistributive instrument to reduce monopsony distortions in the long run. A small minimum wage increases the employment, income, and welfare of initially low-wage workers, but such benefits are exclusively concentrated among them. A large minimum wage can successfully reduce monopsony distortions for higher-wage workers, but at the expense of lowering the employment and income of lower-wage workers in the long run. Appendix E.1 further shows that the aggregate labor market effects of the minimum wage mask this important distributional trade-off.[25]

4.2 Short-Run and Intermediate-Run Effects of the Minimum Wage

Having examined the long-run effects of the minimum wage, we now turn to studying how the economy transitions to a new steady state with a higher minimum. We assume that the minimum wage is unexpectedly introduced starting from the initial steady state. Along this transition path,

[25]Changes in the minimum wage also alter the distribution of aggregate income in the economy across non-college labor income, college labor income, capital income, and firm profits. In particular, the minimum wage transfers resources in both the short and the long run from firm profits to non-college labor income. As shown in Figure E.3 in the appendix, a larger minimum wage increase results in a larger implicit tax on firm profits.

the policy mandates a lower bound $w_{ijt} \geq \underline{w}$ on the period wage that a firm can pay. Given a sequence of intertemporal prices of consumption $\{Q_{t,s}\}$, this constraint on period wages implies a constraint on the present value of wages, which we add to a firm's problem, of the form

$$W_{ijt} \geq \underline{W}_t \equiv \underline{w} + (1-\sigma)Q_{t,t+1}\underline{w} + (1-\sigma)^2 Q_{t,t+2}\underline{w} + \ldots, \tag{22}$$

where $\underline{W}_t$ is the smallest present value of wages consistent with the flow minimum-wage constraint $w_{ijt} \geq \underline{w}$ in each period, and depends on time because the intertemporal prices do. When the minimum wage constraint does not bind for a worker of type i, the first-order conditions are the same as before. When the minimum wage constraint binds, we set $W_{ijt} = \underline{W}_t$ so that a firm's wage offer satisfies (22). Upon the introduction of the minimum wage, firms can fire employed workers; all retained workers must be paid the larger of their existing wage and the minimum wage.[26] All new hires are paid at least the minimum wage in each period.

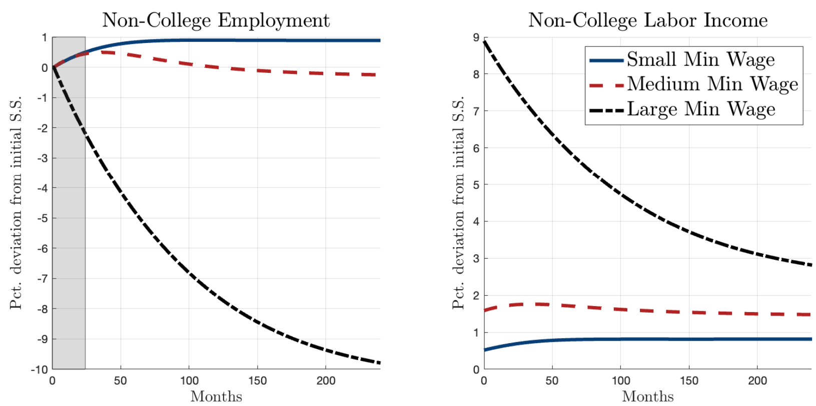
Employment Dynamics. The left panel of Figure 4 shows the transition path of employment for all non-college workers in response to three different minimum wages: \$8.50 (blue line), \$11 (red line), and \$15 (black line). In all three cases, the change in employment is small in the first two years after the increase in the minimum wage: employment increases by 0.5% for the \$8.50 minimum wage, increases even less for the \$11 minimum wage, and decreases by 2% for the \$15 minimum wage. For the minimum wage increases to \$8.50 and \$11, these short-run effects are similar to the ultimate long-run ones. Hence, the short-run effects of small minimum wage increases often estimated in the empirical literature are also informative about the long-run effects, according to our model. However, this is not true for the \$15 minimum wage, in which case *less than one-fifth* of the ultimate employment decline materializes over the first two years. Thus, these results suggest that in general it may be inappropriate to rely on the effects of small minimum wage increases in either the short or the long run to forecast the long-run effects of large minimum wage increases.

Putty-clay adjustment frictions are the key feature that generates the large difference between the short- and long-run effects of a \$15 minimum wage. In general, the difference between putty-clay and standard capital in the employment response to a minimum wage increase is much more

[26]If, instead, we allowed a firm to lower the wage of a worker employed in the initial steady state for whom the minimum wage does not bind, then a firm may have an incentive to renege on an existing wage contract and lower such a worker's wage until the worker is just indifferent between quitting or remaining with the firm. We assume that the original contract that a worker signs with a firm contains a clause that specifies that the firm cannot lower the worker's wage in the event that a minimum wage is introduced. Technically, we imagine that all agents in each period believe that with probability ε a minimum wage will be introduced in the next period and the economy we consider is the limit of such an economy when ε converges to zero.

pronounced for large increases than for small ones. Indeed, as the right panel of Figure 5 shows, in response to a large minimum wage increase, with putty-clay capital, it takes nearly twenty years for aggregate employment to converge to its new steady state, whereas with standard capital, it takes only two years. Instead, as the left panel of Figure 5 illustrates, in response to a small minimum wage increase, with putty-clay capital, it takes nearly eight years for aggregate employment to converge to its new steady state, whereas with standard capital, it takes about four years.

FIGURE 4: Transition Dynamics Following Permanent Increase in Minimum Wage

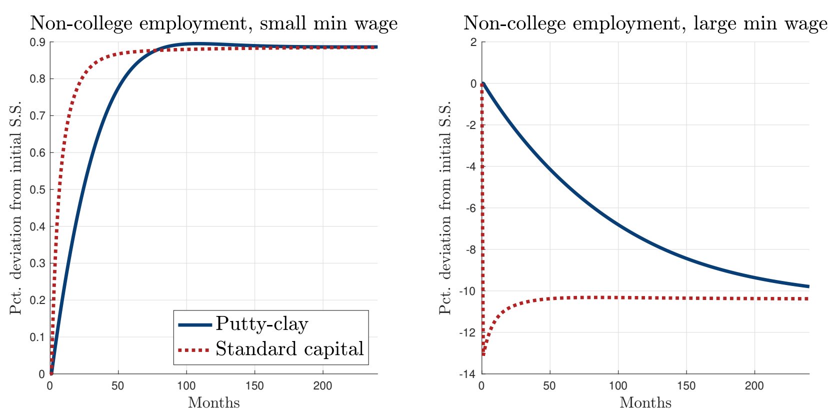


Note: Transition paths of non-college employment (left panel) and labor income (right panel) following the introduction of an \$8.50 (blue line), \$11 (red line), and \$15 (black line) minimum wage.

An intuition for why employment responses between standard and putty-clay capital differ less over time after small increases in the minimum wage than after large ones is as follows. Consider first a small increase in the minimum wage. Since such an increase helps offset monopsony distortions for low-wage workers, a firm with either type of capital wishes to hire *more* low-wage workers. As a result, after a small increase in the minimum wage, a firm finds it profitable to hire more low-wage workers and shift the labor intensity of its capital towards their services accordingly. A firm with standard capital does so instantly. Instead, a firm with putty-clay capital, although it faces the same incentives, adjusts more slowly because it cannot simply replace workers allocated to the existing capital with low-wage ones. Rather, it must purchase *new* capital that is more intensive in these workers' services to be able to absorb them. In general, the smaller the increase in the minimum wage, the smaller the response by either type of firm, and hence the closer the paths of adjustment for firms with the two types of capital.

Consider now the adjustment process in response to a large increase in the minimum wage, above the peak of the long-run Laffer curves of low-wage workers. A firm with either type of capital desires to employ *fewer* of them. Now, a firm with standard capital simply fires many of these workers whose wages have increased above the long-run peaks of their Laffer curves and correspondingly adjusts its capital-labor ratios, while fully utilizing all of its capital. For a firm with putty-clay capital, however, such an adjustment is not optimal: if it fires any workers allocated to the existing capital, the ex-post Leontief structure of production implies it must stop fully utilizing it, thus forgoing part of the profits that this capital would generate. Given the estimated degree of monopsony power, even if the wages of low-productivity workers are increased up to the new minimum, firms continue to employ them because they still earn substantial profits on existing capital—see Figure 1. Hence, a firm with putty-clay capital responds to a large increase in the minimum wage in two ways. First, it continues to operate existing capital at full capacity until it depreciates. Second, it invests in new capital that is less intensive in the services of low-productivity workers, who are now more expensive to employ. This adjustment process takes time, thereby accounting for the large difference in the speed of transition of non-college employment between standard and putty-clay capital in the right panel of Figure 5.

FIGURE 5: Role of Putty-Clay Frictions in Transition Dynamics



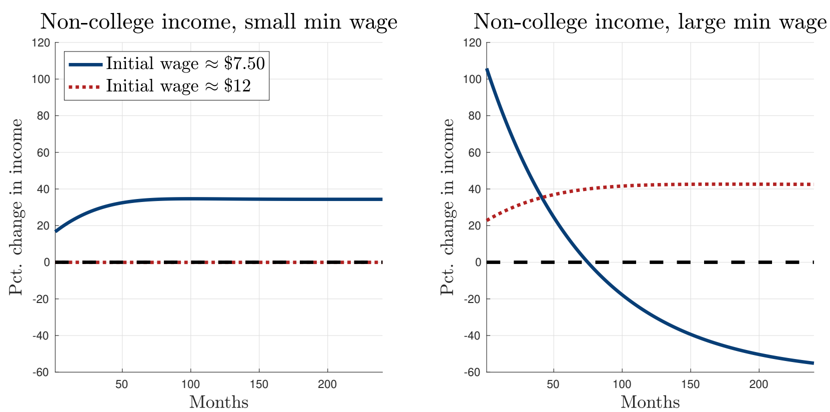
Note: Transition path following the introduction of an \$8.50 minimum wage (left panel) and a \$15 minimum wage (right panel) with putty-clay capital (blue line) and standard putty-putty capital (red line). *Putty-clay* refers to the baseline model. *Standard capital* refers to the version of the model with standard capital.

Figures E.5 and E.6 in Appendix E further illustrate these mechanisms by displaying the distribution of capital types along the transition path to the new steady state with a high minimum wage. Figure E.4 further shows that these dynamics upon impact generate a near point mass in the wage distribution at the new minimum wage, reproducing the typical bunching of empirical wage distributions. This near point mass shrinks along the transition to the new steady state, as firms substitute away from low-productivity workers, although some bunching persists as in the data.

Ultimately, we view putty-clay frictions as a parsimonious way to capture a rich set of frictions to firms adjusting their *entire* input mix. As long as firms face some adjustment costs to altering their production setup, the short-run employment response to large minimum wage increases will markedly differ from the long-run response. In our framework, the speed of the employment dynamics is primarily governed by the depreciation rate of capital δ, because δ determines the rate at which the old, labor-intensive capital is replaced by new, less labor-intensive capital. Our baseline assumes a depreciation rate of 15% to be consistent with aggregate NIPA data on equipment, but our results are robust to using a 10% rate that includes structures; see Figure E.7 Appendix E.

Labor Income Dynamics. The right panel of Figure 4 shows the transition dynamics for the labor income of non-college workers when the minimum wage is raised to \$8.50, \$11.00, and \$15.00. In all cases, labor income increases in the short run because the minimum wage raises wages but employment is not yet affected much. For the small- and medium-sized minimum wage increases, these income gains persist over the transition path because employment does not substantially change overall. For the large minimum wage increase, however, employment gradually falls over time, causing the increase in labor income to dissipate as well.

Figure 6: Labor Income for Two Non-College Workers in Response to Minimum Wage



Note: Flow labor income $w_{\ell z,t} n_{\ell z,t}$ for two types of non-college workers along the transition path following the introduction of an \$8.50 minimum wage (left panel) and a \$15 minimum wage (right panel).

The two panels of Figure 6 delve deeper into the micro-level dynamics of labor income following a small (\$8.50) and a large (\$15) minimum wage increase. In the left panel, we contrast the time paths of labor income for a non-college worker initially earning \$7.50 (blue line) and another one initially earning \$12 (red line), starting from the initial steady state. In response to the small minimum wage increase, workers initially earning \$7.50 experience an increase in labor income of about 20% upon impact. Most of this increase follows from the increase in their wage from \$7.50 to the new minimum of \$8.50, but some of it occurs because employment increases as well over time due to the reduction in monopsony distortions. By contrast, there is no such effect on workers initially earning about \$12 per hour, because they are paid far above the new minimum.

The \$15 minimum wage generates more dramatic changes in labor income over time for both types of workers. In the short run, such a minimum nearly doubles income for workers initially earning \$7.50 with almost no impact on their employment. Over time, however, firms substitute away from these workers, leading their total labor income to fall. After about ten years, such workers' income falls below its initial level and remains low forever after. Instead, labor income monotonically increases for workers initially earning \$12 because the minimum wage reduces monopsony distortions for these workers, inducing firms to hire more of them.

These results highlight how a large minimum wage generates short-run benefits but long-run costs for low-wage workers. The short-run benefits arise because putty-clay frictions discourage firms from immediately substituting away from the lowest-productivity workers for whom the minimum wage binds. However, once firms shift their capital stock away from such workers, employment declines, which leads to large long-run costs. Cohorts of the lowest-paid workers who start their careers after the introduction of the new minimum wage enjoy less of the short-term benefits and bear more of the long-term costs—the more so, the later these cohorts enter.

4.3 Welfare Effects

We now study how the minimum wage affects welfare, taking into account all its short-run benefits and long-run costs. Define the *dynamic welfare change* as the value of Δ_i that solves

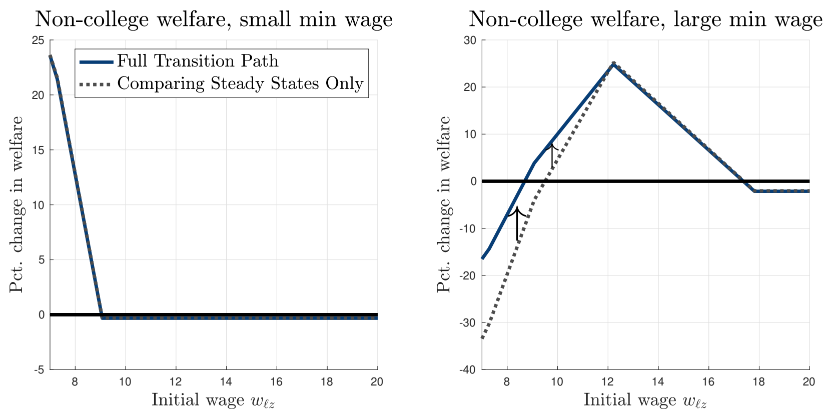
$$\textstyle\sum_{t=0}^{\infty}\beta^t u((1+\Delta_i)c_i^*, n_i^*, s_i^*) = \sum_{t=0}^{\infty}\beta^t u(c_{it}, n_{it}, s_{it}), \tag{23}$$

where x^* denotes the value of any variable x in the initial steady state and the right-hand side of (23) is the value of utility along the entire transition path. Here, Δ_i measures the percentage change in consumption at the initial steady state that would make a household indifferent between the initial steady state and transitioning from it to a final steady state with a higher minimum wage. Hence, Δ_i is positive if the minimum wage increase makes a household of type i better off and negative otherwise.[27] Figure 7 plots these dynamic welfare effects (solid line) as well as the

[27] Welfare differs from labor income because consumers enjoy the benefits of leisure and experience disutility when working and searching for jobs. Note that our welfare analysis depends on the distribution of profits across households. We assume that profits are distributed in proportion to each households' share of total labor income.

change in welfare excluding the transition dynamics (dashed line)—this latter welfare measure is calculated only relative to the new long-run steady state.

Figure 7: Welfare Effects of Minimum Wage



Note: Welfare change due to an \$8.50 minimum wage (left panel) and a \$15 minimum wage (right panel). The blue line plots welfare along the transition path computed as $\widetilde{\Delta}_i$ from (23). The gray line plots welfare changes comparing only steady states. The x-axis corresponds to the wage $w_{\ell z}$ of a type-z worker in the initial steady state.

In response to a small minimum wage of \$8.50 (left panel), the static and dynamic welfare changes are essentially identical because the economy transitions fairly quickly to the new steady state—recall Figure 4. However, the two measures differ in response to a large minimum wage of \$15 (right panel). The steady-state welfare comparison substantially *overstates* the welfare losses of the minimum wage for low-productivity workers because it ignores their short-term gains in terms of higher labor income early in the transition, as described above. In fact, after accounting for the large short-run welfare gains, only 6% percent of the current cohort of non-college workers are made worse off after the introduction of a \$15 minimum wage.

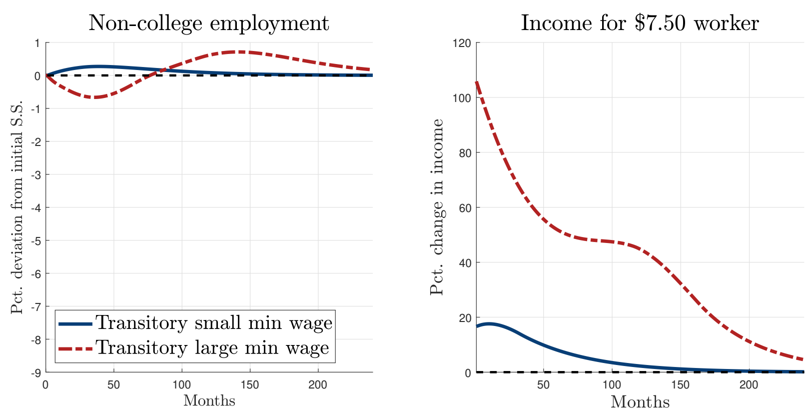
4.4 Temporary Changes in the Minimum Wage

Although so far we have considered only permanent increases in the real minimum wage, the minimum wage is set in nominal terms in the United States, so the real minimum declines between legislated changes due to general price inflation. Productivity growth also implies that the real value of the minimum wage decreases relative to average economy-wide wages over time.

Here we consider a simple example in order to examine the implications of these dynamics in the periods between legislated changes in the nominal minimum wage. We model a temporary increase in the real minimum wage through a path for it that decays at constant rate g such that $\underline{w}_t = \underline{w}_0(1+g)^t$. The decay rate g corresponds to the sum of inflation and real productivity growth over time. For illustration, we consider a value of g equal to 5% annually, which we interpret as corresponding to 2% productivity growth and an inflation rate of 3% that accounts for baseline inflation plus any additional inflation induced by the minimum wage.

The left panel of Figure 8 shows two important differences in how employment responds to a temporary increase in the minimum wage, compared to the permanent increases studied above. First, the overall response to a temporary increase is an order of magnitude smaller than the response to a permanent one. This difference is partly due to the fact that the present value of a transitory minimum wage increase is smaller than the present value of a permanent minimum wage increase. But even in the early phase of the transition, when the real value of the minimum wage in the two scenarios is relatively more similar, putty-clay frictions induce firms to reduce the employment of low-productivity workers by much less in the transitory case than in the permanent case. Intuitively, firms anticipate that any new capital they install early on in the transition will still be in use later on, when the decaying minimum wage will no longer bind for lower-productivity workers, who will then be once again desirable to employ.

Figure 8: Transition Dynamics Following Temporary Increase in Minimum Wage



Note: Transition paths following the introduction of a time-varying minimum wage $\underline{w}_t$ that decays at rate 5% per year. Left panel plots the time path of employment for all non-college workers for an $8.50 (small) and a $15.00 (large) temporary minimum wage increase. Right panel plots the time path of labor income for a worker initially earning $7.50 in response to a small and a large temporary minimum wage increase.

The second difference between the permanent and transitory minimum wage paths is that the temporary path features non-monotonic dynamics. For instance, after a large minimum wage increase, employment immediately declines below its initial level, but it then increases above it before reverting back to its steady-state level. This occurs because the real value of the minimum wage is initially relatively high—above the efficient level of wages for sufficiently many workers to decrease employment—but later declines to a relatively low level—close to the efficient level of wages for sufficiently many workers to increase employment.

The right panel of Figure 8 compares the paths of labor income for the lowest-wage workers in response to a small and a large temporary increase in the minimum wage. After the introduction of a \$15 minimum wage, which doubles these workers' initial wage of \$7.50, workers' labor income increases by 100%. These income gains decline as the real minimum wage declines over time. Interestingly, the labor income of such workers is strictly higher in every period under a large temporary minimum wage increase than under a small one. That is, a one-time unanticipated introduction of a decaying large minimum wage allows low-wage workers to reap the short-run benefits of a large minimum wage without incurring its long-run costs.

We use this transitory minimum wage experiment to illustrate the mechanisms of our model and the importance of agents' beliefs about the permanence of any policy change, rather than to suggest that this is a tool that policymakers could repeatedly exploit. A full analysis of such a scenario would require carefully modeling agents' expectations about the duration of policies.

5 Short-Run and Long-Run Effects of the EITC

A major drawback of the minimum wage is that it is too blunt a redistributive instrument to support the labor income of all workers earning low wages in the long run. Other policies within the U.S. tax and transfer system, which are better targeted to the population of interest, may be more effective at redistributing resources to a larger group of workers, including those at the low end of the wage distribution, who are the intended beneficiaries of these policies. Here, we consider one such policy, the EITC, which amounts to a subsidy paid by the government to working families and constitutes one of the largest components of transfer payments in the United States. We show that such a policy has larger long-run labor market benefits for low-wage workers than a comparable increase in the minimum wage. Additionally, the EITC is more beneficial in the long run than in the short run and thus displays the opposite time profile of benefits for low-wage workers, compared with a large minimum wage. Namely, by subsidizing the employment of workers earning low incomes, the EITC induces firms to substitute towards such workers rather than away from them, as the minimum wage does. That is, faced with the EITC, firms adjust their mix of inputs by investing in capital that is more intensive in the use of the labor services of these workers.

5.1 An Overview of the EITC

We introduce the EITC in our model by incorporating the transfer schedule illustrated in Figure F.1 in Appendix F, which reproduces the main features of the actual EITC and entails three regions: *i*) a *phase-in* region, in which the subsidy is proportional to household income (left panel) with a positive marginal subsidy rate of 25% (right panel); *ii*) a *plateau* region, in which the subsidy is capped at its maximum benefit with a marginal subsidy rate of zero; and *iii*) a *phase-out* region, in which the subsidy is reduced proportionally to any additional income with a negative marginal subsidy rate of 22%. In the phase-in region, households face both a positive average subsidy rate, since the total tax credit is positive, and a positive marginal subsidy rate, since the credit is being phased in. In the plateau region, households still face a positive average subsidy rate, since they continue to receive the transfer, but now face a zero marginal subsidy rate. In the phase-out region, households face a positive average subsidy rate but a negative marginal subsidy rate or, equivalently, a positive marginal tax rate because each additional dollar of earnings reduces their transfers.

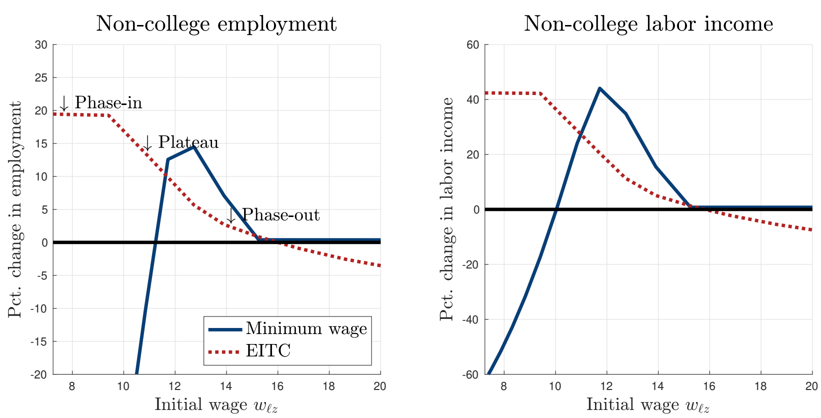
To adequately contrast the effects of the minimum wage to those of the EITC, we make the two policies budget equivalent. Specifically, we assume that the EITC described is financed through a linear tax on firm profits such that the revenue from the tax equals the loss in profits associated with the minimum wage that the EITC is being compared with. That is, we choose an *explicit* tax on profits to fund the EITC that is equal to the *implicit* tax on profits induced by the minimum wage. We assume that both investment and vacancy posting costs are fully deducted from profit taxes, which implies that the profit tax does not distort firms' decisions. We choose the parameters of the EITC schedule to ensure this form of budget equivalence. Hence, both the minimum wage and the EITC transfer the same total amount of income from firms to households: the two policies only differ in how they distribute those resources across households.

5.2 Long-Run Effects of the EITC

Mirroring our analysis of the minimum wage, we first study the long-run effects of the EITC and then examine the transition dynamics of the economy in response to it. We discuss the complementarities between the minimum wage and the EITC in Section 8. In the long run, the EITC works in opposite ways compared with a large increase in the minimum wage. Whereas such a minimum wage eventually induces firms to substitute *away* from low-productivity non-college workers, as they become relatively more expensive to employ, the EITC incentivizes firms to substitute *towards* them by subsidizing their employment. Eventually, firms alter their input mix so as to employ more of these workers by investing in capital that is more intensive in the use of their labor services. The EITC also increases the long-run employment, labor income, and welfare of non-college workers by stimulating job creation, since it partially offsets the monopsony distortions to firms' incentives to create jobs whenever the EITC marginal subsidy rate is positive, as discussed in Section 2.

Figure 9 illustrates the interplay between these effects. The red lines in the figure show how the EITC affects the long-run employment (left panel) and labor income (right panel) of non-college workers. The blue lines in the figure also reproduce how a minimum wage of $15 affects the long-run employment (left panel) and labor income (right panel) of non-college workers. Two features emerge. First, comparing the red to the blue lines in the figure reveals that the EITC is much more effective than the minimum wage at stimulating the employment and labor income of initially low-wage non-college workers in the long run—the vertical difference between the red and the blue lines is largest at low levels of initial wages. Second, the distributional effects of the EITC are very different across workers earning different initial wages.

FIGURE 9: Long-Run Effects of EITC on Non-College Employment and Labor Income



Note: Percentage change in steady-state employment (left panel) and labor income (right panel) of select z-types among non-college workers under an EITC budget-equivalent to a $15 minimum wage, relative to the initial steady state. The x-axis is the initial wage $w_{\ell z}$ of a z-type. *Minimum wage* plots the effect of a $15 minimum wage.

To elaborate, note that the lowest-wage workers experience the phase-in portion of the policy and hence enjoy both the beneficial effect of the EITC average subsidy on their wages and of the EITC marginal subsidy, which is positive, on firms' job creation decisions. Thus, the EITC increases the wages of low-productivity workers, the number of jobs targeted at them, and their labor income. As a result, low-productivity workers benefit the most from the policy in terms of their employment and labor income, as the two panels of Figure 9 show. Overall, the EITC leads to an increase in non-college employment by 5.9%, which is smaller than the gain experienced by workers initially earning low wages for two reasons—the impact of the EITC on participation (not shown) is similar. First, in the phase-out region in which the marginal subsidy rate is negative, the EITC acts as a tax on job creation reducing employment. Second, the subsidy eventually dies out and so it does not affect any worker initially earning more than $16.

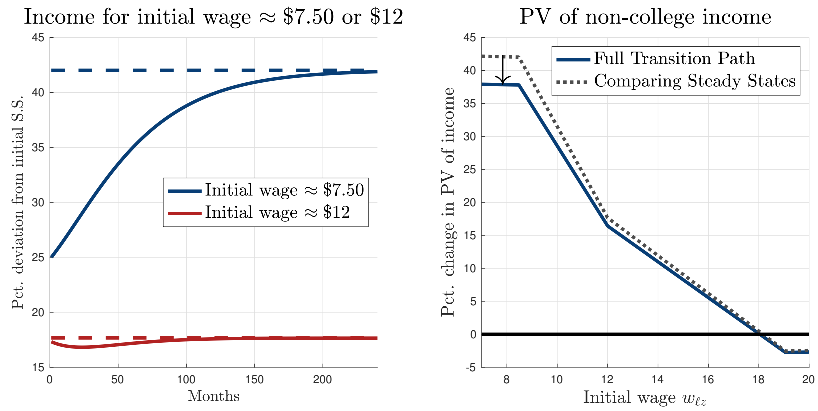
Why does the EITC generate opposite substitution incentives for firms compared with the minimum wage? The minimum wage directly raises the wage of initially low-wage workers and so firms' cost of hiring them. When this increase is sufficiently large, their wage will exceed the efficient level, inducing firms to substitute away from such workers and so reducing their employment. With the EITC, instead, the government pays the marginal cost of increasing the wage of workers at the low end of the wage distribution. In addition, a portion of the subsidy paid to workers is appropriated by firms in the form of lower before-transfer wages—after-transfer wages can be higher with the subsidy than without it, even if before-transfer wages are lower. This indirect subsidy to firms in turn induces them to adjust their production mix towards low-wage workers, which leads to an increase in their employment in the long run.

5.3 Short-Run and Intermediate-Run Effects of the EITC

We now consider the transition path induced by the EITC. The left panel of Figure 10 plots the time profile of the labor income of two types of non-college workers, earning $7.50 and $12 before the policy is introduced—the dashed lines denote the long-run values of income for the two types of workers. Consider workers initially earning $7.50. Their after-transfer wage immediately increases after the introduction of the EITC due to the subsidy the EITC entails, and then continues to rise over time, as firms increasingly hire more of these workers. As in the case of the minimum wage, this adjustment occurs gradually due to putty-clay frictions, as firms let existing capital depreciate and only then replace it with new capital that uses the labor services of such workers more intensively. In particular, these frictions delay the long-run *benefits* of the EITC, just as they delayed the long-run *costs* of a large minimum wage increase. Thus, stark differences emerge in the time profile of the benefits and costs of the two policies with putty-clay capital.

The right panel of Figure 10 illustrates the importance of accounting for the intertemporal path of the benefits and costs of the EITC. It does so by contrasting the change in the present value of labor income of non-college workers, computed along the entire transition path, with the change in the present value of their labor income in the new steady state, after the EITC is introduced. It is apparent that because of putty-clay frictions, comparing steady states would lead to an overstatement of the benefits of the EITC—these patterns are the *opposite* of those implied by the minimum wage. Appendix F shows that results are qualitatively similar for a smaller-sized EITC that is budget-equivalent to a \$10 minimum wage.

FIGURE 10: Dynamic Effects of EITC



Note: Dynamic effects of introducing the EITC starting from the initial steady state without it. Left panel plots labor income for two types of non-college workers. Right panel plots the change in the present value of income among non-college workers: the blue line computes the present value over the entire transition path and the black line only compares steady states. The x-axis is the initial wage $w_{\ell z}$ of a z-type worker.

6 Role of Monopsony Power and Labor-Labor Substitutability

Here we present a number of robustness exercises to explore the role of firm monopsony power, captured by ω, and the degree of input substitution, captured by ϕ, in determining the labor market responses to the minimum wage and the EITC in both the short and the long run.[28]

Long-Run Employment. Consider first the minimum wage. Table 5 shows how the long-run employment effects on non-college workers of various-sized minimum wage increases depend on the degree of firm monopsony power and the elasticity of substitution across workers.[29] The left panel examines the employment effects of a small (permanent real) minimum wage increase to \$8.50, whereas the right panel examines those of a large minimum wage increase to \$15. Recall that in our baseline parameterization we target a wage markdown of 0.75 and ϕ equals 4.

[28]In Appendix G, we show the robustness of our policy results to changes in other parameters such as α (the elasticity of substitution between capital and the aggregate labor bundle), ρ (the elasticity of substitution between the bundles of college and non-college workers), and γ_n (the utility parameter that is a key determinant of households' labor supply elasticity). Changes in these parameters, compared to changes in the parameters ω and ϕ, have a much more muted effect on our quantitative results. We also show that changes in the parameters that govern the search part of our model (κ_0, η, and σ) have only small effects on our key results.

[29]We reparameterize the model for each of the different values for ϕ and targets for the wage markdown.

As shown in the table, the long-run employment effects of a minimum wage increase of any size become more negative when either firm monopsony power is lower, for instance, when the targeted wage markdown increases to 0.9, or labor-labor input substitutability within an education group is higher, for instance, when ϕ increases to 6. Intuitively, when firm monopsony power is weaker, there is less room for minimum wage increases to induce efficient employment gains. Likewise, when labor-labor substitutability is higher, firms more readily substitute away from workers whose marginal product is lower than the minimum wage in the long run. As evident from both panels of the table, long-run employment declines are more pronounced for the larger minimum wage increase across the different values for the targeted wage markdown and ϕ.

Table 5: Robustness of Non-College Employment to Small and Large Minimum Wage

	Small Minimum Wage in the Long Run			Large Minimum Wage in the Long Run		
	Targeted Wage Markdown			Targeted Wage Markdown		
ϕ	0.6	0.75	0.9	0.6	0.75	0.9
2	2.3%	1.8%	1.3%	-0.1%	-6.3%	-6.4%
4	1.9%	0.8%	-2.3%	-8.9%	-12.1%	-17.4%
6	-0.1%	-2.7%	-3.2%	-14.4%	-19.4%	-20.0%

Note: Total employment change for non-college workers in response to an \$8.50 minimum wage (left panel) and a \$15 minimum wage (right panel) for various values of ϕ and the wage markdown. The baseline value of ϕ is 4 and that of the wage markdown is 0.75.

Table 6: Robustness of Non-College Employment to Small and Large EITC

	Small EITC in the Long Run			Large EITC in the Long Run		
	Targeted Wage Markdown			Targeted Wage Markdown		
ϕ	0.6	0.75	0.9	0.6	0.75	0.9
2	0.1%	0.1%	0.1%	2.0%	4.1%	4.7%
4	0.3%	0.3%	0.4%	4.4%	5.9%	5.7%
6	0.5%	0.6%	0.6%	5.4%	6.2%	6.3%

Note: Total employment change for non-college workers in response to the introduction of the EITC for various values of ϕ and the wage markdown. *Small EITC* is budget-equivalent to an \$8.50 minimum wage. *Large EITC* is budget-equivalent to a \$15 minimum wage. The baseline value of ϕ is 4 and that of the wage markdown is 0.75.

Analogously, Table 6 shows how the degree of firm monopsony power and the elasticity of substitution across workers affect the long-run employment responses to the introduction of an EITC that is budget equivalent to an \$8.50 minimum wage (left panel) and a \$15 minimum wage (right panel). Namely, we assume that the EITC is funded by a tax on firm profits equal to the profit loss from a small (\$8.50) or a large (\$15) minimum wage in our baseline parameterization, keeping its size fixed at either level throughout the experiments we perform. Not surprisingly, the long-run effects of the EITC on employment are of *opposite* sign compared to those of either a small minimum wage increase, when the degree of firm monopsony power is small or of labor-labor substitution is large, or a large minimum wage increase. Under the EITC, as monopsony power decreases, that is, as the wage markdown increases, or as the elasticity of substitution among workers increases, non-college employment increases for two reasons. First, since the EITC helps correct a smaller monopsony distortion, it induces firms to hire non-college workers initially earning low wages at close to the efficient level. Second, as workers become more substitutable, a given EITC subsidy for low-productivity workers leads firms to alter their production mix towards them relatively more.

Short-Run Elasticities of Employment Response: Model vs. Data. Here, we contrast short- and medium-run employment elasticities to the minimum wage and the EITC implied by our model and compare them to the their estimates in the literature.

Consider first the short-run responses to (permanent real) minimum wage increases of various sizes implied by our model under different parameterizations.[30] To facilitate the comparison with their estimates in the literature, we report in Table 7 the elasticities of the response of non-college employment to increases in the minimum wage by \$1.25 to \$8.50 in the left panel and by \$2.50 to \$9.75 in right panel, since most of the empirical literature has explored increases in this range. As the literature has mostly studied the short- and medium-run impact of minimum wage increases, we use our model to examine the labor market effects of these increases at horizons ranging between one and four years. Finally, given that the empirical literature has often focused on low-wage workers, we illustrate the employment elasticities implied by our model for the group of workers earning less than \$15 prior to any minimum wage increase.

Table 7: Robustness of Non-College Employment Elasticities to Minimum Wage

	\$1.25 increase				\$2.50 increase			
	1 year	2 years	3 years	4 years	1 year	2 years	3 years	4 years
Baseline	0.01	0.03	0.03	0.04	0.02	0.02	0.02	0.02
Larger markdown (0.65)	0.03	0.05	0.06	0.07	0.05	0.08	0.10	0.12
Smaller markdown (0.85)	-0.02	-0.03	-0.04	-0.05	-0.02	-0.04	-0.05	-0.07
Higher substitutability ($\phi = 6$)	-0.06	-0.06	-0.05	-0.05	-0.15	-0.15	-0.15	-0.16

Note: Elasticities of non-college employment with respect to the minimum wage at various horizons. Left panel computes elasticities in response to a \$1.25 increase in the minimum wage and right panel in response to a \$2.50 increase in the minimum wage. *Baseline* refers to the baseline parameterization. *Larger markdown* refers to a re-parameterized model targeting a wage markdown of 0.65 (more monopsony power). *Smaller markdown* refers to a re-parameterized model targeting a wage markdown of 0.85 (less monopsony power). *Higher substitutability* refers to a re-parameterized model under the assumption of a higher labor-labor substitutability ($\phi = 6$).

Table 7 shows these elasticities for different values of the wage markdown and labor-labor substitutability. Note that across the range of parameter values considered, the short- and medium-run employment elasticities to minimum wage increases are always small in absolute value, but their sign depends on the values of our key parameters. For instance, the employment effects of minimum

[30] Many empirical studies exploit regional variation in minimum wages, which may not be well reflected in our general equilibrium setup, in which capital markets clear. We have alternatively studied a small-open-economy version of our model in which the intertemporal prices of consumption $\{Q_{t,s}\}$ are taken as given and found very similar results.

wage increases are more likely to be negative in the short and medium run when the wage markdown is relatively small (say, 0.85) or labor-labor substitutability is relatively large (say, $\phi = 6$).

Table 8 exhibits the response of non-college employment to an EITC that is budget-equivalent to a \$15 minimum wage in our baseline. As in Table 6, we keep the same EITC schedule across the parameterizations considered. Note that for each parameter combination, the EITC increases non-college employment. However, because of our putty-clay frictions, the short-run effects one or two years after the introduction of the EITC are substantially smaller than the long-run ones.

Table 8: Robustness of Non-College Employment to EITC

	1 year	2 years	3 years	4 years
Baseline	1.6%	2.7%	3.5%	4.0%
Larger markdown (0.65)	1.4%	2.3%	3.0%	3.5%
Smaller markdown (0.85)	2.0%	3.3%	4.3%	4.9%
Higher substitutability ($\phi = 6$)	1.9%	3.2%	4.0%	4.6%

Note: Percentage change in non-college employment in response to an EITC that is budget-equivalent to a \$15 minimum wage at various time horizons. *Baseline* refers to the baseline parameterization. *Larger markdown* refers to a re-parameterized model targeting a wage markdown of 0.65 (more monopsony power). *Smaller markdown* refers to a re-parameterized model targeting a wage markdown of 0.85 (less monopsony power). *Higher substitutability* refers to a re-parameterized model under the assumption of a higher labor-labor substitutability ($\phi = 6$).

As the last line of Table 8 shows, the employment gains for non-college workers increase with the degree of substitutability of workers relative to our baseline, uniformly across horizons. Intuitively, since the EITC subsidizes the employment of low-productivity workers, it induces firms to hire more of them over time, as firms replace the existing capital with new capital that is more intensive in the use of their labor. This process leads to larger intermediate-run responses, which is broadly consistent with the empirical literature (see, for instance, Meyer and Rosenbaum (2001)).[31]

7 Accounting for the Empirical Evidence

Here we discuss how our framework can be used to interpret and possibly inform empirical research. We start by highlighting how it can match the wide range of empirical estimates of the short- and medium-run labor market responses to increases in the minimum wage. According to our model, the magnitude of employment and income responses depends on *i*) the size of the minimum wage increase; *ii*) the degree of input substitutability at the firm level; *iii*) the extent of firm monopsony power in the relevant labor market; and *iv*) the permanence of the policy change. Variation in any of these margins across regions or time can explain the heterogeneity of estimates in the literature.

[31] Kleven (2019) argues that most of the positive effects of the EITC are driven by its expansion in 1993, whereas more modest expansions of it do not lead to any sizeable change in employment. In our model, the EITC may lower employment among some groups of workers if the phase-out rate implies a large enough increase in households' marginal tax rate, but the strength of this effect depends on the tax and transfer system in place. We explore some of these policy interactions in Section 8.

We then discuss the conditions under which estimates of the short-run labor market responses to changes in policy can be useful in forecasting long-run employment and earning responses.

7.1 Empirical Support for Model Predictions

Our model predictions are consistent with a wide range of empirical estimates from the literature. For example, Neumark and Shirley (2022) recently reviews the empirical literature on the minimum wage by conducting a meta-analysis of 109 published studies based on cross-state variation in the minimum wage in the United States and reporting the implied elasticity of the employment response. Most of the literature focuses on the short-run effects of relatively small minimum wage increases. Essentially all of the papers in the survey examine the employment effects of increases of $3 or less over relatively short time periods of one or two years. Also, all these papers tend to focus on the employment effects for lower-earning workers, such as teenagers and young adults, who are most likely to be effected by changes in the minimum wage.

Neumark and Shirley (2022) emphasizes that roughly 80% of the studies they review find zero to small short-run employment declines in the two years after a minimum wage increase—the rest of the studies document small positive increases. Our results in Table 7 show that relatively small decreases in the extent of firm monopsony power, as captured by the wage markdown, or increases in the degree of labor-labor substitutability, as captured by ϕ, relative to our baseline can generate small *negative* employment responses to small minimum wage increases in the short to intermediate run. Likewise, relatively small increases in the extent of firm monopsony power or decreases in the degree of labor-labor substitutability relative to our baseline can generate small *positive* employment responses to small minimum wage increases in the short to intermediate run. Overall, we find it encouraging that our model can replicate the range of small short- and intermediate-run employment responses to minimum wage changes estimated in the literature.

Clemens and Strain (2021) provides evidence of a differential employment response to minimum wage increases of different sizes at different horizons. Specifically, they estimate the employment effects of small (less than $2.50) and large (more than $2.50) state-level increases in the minimum wage in both the short run (1 to 3 years after a minimum wage increase) and the medium run (4 to 6 years after a minimum wage increase). These authors find that in both the short and the medium run, small and large increases in the minimum wage have insignificant effects on employment. In the medium run, only large minimum wage increases have statistically significant, and negative, effects on employment. As shown in Figure 4 and Table 7, our model replicates these findings as well. In the short run, employment responds little to both small and large minimum wage increases regardless of the value of the model parameters. However, at longer horizons, the employment response becomes larger in absolute value, as the size of the minimum wage increase becomes bigger—for small minimum wage increases, short- and long-run responses are roughly similar. We take it as a strength of our framework that we can match the dynamic employment responses to minimum wage increases documented by Clemens and Strain (2021).

In a recent paper, Cengiz et al. (2019) develops a novel empirical strategy to estimate both the short- and the long-run responses to small minimum wage increases. Two features of the approach in Cengiz et al. (2019) are especially relevant in the context of our analysis. First, the minimum wage increases they explore are *small*—averaging across all the increases they study, the minimum wage was raised by about 10% or about $0.75 in current dollars. Second, Cengiz et al. (2019) explore changes in employment up to seven years after a minimum wage increase. We interpret these results as estimating effectively the long-run effects of a small minimum wage increase. The authors find that employment effects are small and positive in the few years after a small minimum wage increase, which persist over a seven-year horizon—the employment increases they detect, though, are not statistically different from zero. Cengiz et al. (2019) then conclude that there does not seem to be any significant labor-labor substitution in response to small minimum wage increases up to seven years after an increase is introduced. As apparent from the left panel of Figure 4, our framework qualitatively replicates these findings as well. Specifically, in response to a minimum wage increase of about $2, non-college employment increases only slightly after the imposition of the new minimum and remains roughly constant seven years after such an increase.

A recent literature also provides evidence that firms slowly adjust their input mix in response to minimum wage increases, in line with the predictions of our model. For example, Meer and West (2016) estimates that an increase in the minimum wage reduces employment but such an effect takes several years to materialize. In response to a large and persistent minimum wage increase in Hungary, Lindner and Harasztosi (2019) documents that firms responded by substituting away from labor towards capital. Clemens, Kahn and Meer (2021) finds that U.S. firms substitute away from low-productivity workers towards higher-productivity ones in response to minimum wage increases.

Overall, our framework can reproduce many of the key empirical findings about the employment effects of minimum wage increases of various magnitudes across different time periods.

7.2 Implications of Our Model for Empirical Literature

We now turn to discussing how our model can help interpret existing empirical findings and inform the implementation of empirical work.

Wide Range of Empirical Estimates is Possible Depending on Fundamentals. An open debate within the labor economics literature concerns the sign and size of the employment response to minimum wage increases. As discussed, the bulk of the literature documents small, either positive or negative, short-run employment effects of small to moderate increases in the minimum wage (Neumark and Shirley (2022)). We showed above that all of these estimates are plausible, depending on the specific features of the labor markets considered. For example, settings with slightly less firm monopsony power or slightly more labor-labor substitutability tend to lead to small *negative* short-run employment responses to minimum wage increases. Conversely, settings with slightly more firm monopsony power or slightly less labor-labor substitutability tend to lead to small *positive* short-run employment responses to minimum wage increases. Hence, through the lens of our model, interpreting or comparing the labor market effects of minimum wage increases requires taking a stand on the value and comparability of labor market fundamentals such as the extent of firm monopsony power and the degree of input substitutability in production.[32]

Extrapolating From Short-Run Responses to Learn About Long-Run Responses. To date, the empirical literature as a whole has provided little guidance on the *long-run* labor market impact of *large* policy changes. In the context of the minimum wage, this lack of empirical evidence has been pointed out, for instance, by Brown (1999), which states that "*[t]here is simply a stunning absence of credible evidence—indeed, of credible attempts—[to identify] the long run effects [of the minimum wage].*" One appeal of our framework is that it allows us to make predictions about the long-run effects of large minimum wage increases or large expansions of the EITC, given assumptions about the extent of firm monopsony power in the labor market and the degree of input substitutability. Indeed, our model suggests some rules of thumb for when short-run employment and income responses to policy changes are good predictors of potential long-run responses.

For example, as shown in Figure 4, moderate increases in the minimum wage have short-run employment and labor income effects that are fairly similar to their ultimate long-run effects. Likewise, as illustrated in Figure 10, the EITC has short-run employment and labor income effects that are similar to their ultimate long-run effects. These results highlight that for small to moderate

[32]Part of the literature debate focuses on the econometric strategies to best estimate the effects of interest in the first place. Our work is silent about this component of the debate.

increases in the minimum wage and for the EITC, researchers should feel comfortable using short-run labor market responses to forecast the corresponding long-run ones.

In general, however, this is not necessarily the case, as we show in Figure 4 for a large increase in the minimum wage. As apparent from the figure, only about 20 percent of the ultimate long-run impact of a $15 minimum wage on the employment of non-college workers occurs in the first two years. Moreover, for workers earning the lowest wages, the short-run labor income response is small and *positive* whereas the long-run response is large and *negative*. These findings suggest that it may be unwarranted to extrapolate from short-run responses to either small or moderate minimum wage increases to infer the long-run impact of large increases.

Accounting for Duration of Policy Changes. As most actual minimum wage policies are set in nominal terms for extended periods of time, our analysis in Figure 8 makes it clear that before inferring the impact of such policies from any evidence about it, one must explicitly take a stand on the extent to which firms and consumers *expect* the policy to be temporary or permanent in real terms. This requires also taking a stand on the *beliefs* of these agents with respect to future inflation and productivity growth, which affect the degree of permanence of these policies in real terms. Otherwise, the interpretation of any estimated labor market effects is ambiguous. For instance, a small positive employment response in the medium-run to a minimum wage increase could be due to: *i*) the change being thought to be permanent and monopsony power in the relevant market to be large relative to the size of the minimum wage increase; or *ii*) the change being thought to be permanent and the substitutability across inputs in production to be low; or *iii*) the change to be thought to be temporary, regardless of the degree of monopsony power and input substitutability. A key insight of our framework is that if researchers wish to extrapolate their findings to other settings or time periods, it is important to distinguish among these features of labor markets, which we argue are at the core of any estimated employment effects. For example, our results suggest that it is inappropriate to extrapolate the labor market effects of temporary increases in the real minimum wage to forecast the labor market effects of similarly-sized permanent increases.

Extrapolating From Regional Estimates to Learn About National Estimates. Many of the existing estimates of the labor market effects of the minimum wage are based on studies that exploit cross-regional variation in the minimum wage within the United States.[33] One needs to proceed with caution when extrapolating these findings so as to infer the impact of changes in the federal minimum wage. The reason is that it is likely that some combination of monopsony power

[33]See, for example, Card and Krueger (2015) and the recent survey by Neumark and Shirley (2022).

and local labor-labor substitutability varies across localities within the United States. If so, then the estimates of effects from any one regional or cross-regional study would not be able to precisely predict effects at the national level, which are presumably close to a weighted average of the level effects in each region.[34] What is comforting, however, is that for a broad range of parameter values, our model robustly reproduces the relatively small short-run elasticities of response of employment estimated in the literature. So, it may well be justified to deduce from values in the mid-range of a large number of regional studies of the impact of small to moderate increases in the minimum wage a reasonable range of potential impacts of small to moderate increases at the federal level.

8 Combining Minimum Wage, EITC, and General Tax Policies

Having considered the effects of the minimum wage and the EITC in isolation, we now explore how the two policies may interact. We argue that moderate increases in the minimum wage complement both the EITC and a more general non-linear tax system that captures the main features of the U.S. tax and transfer system. Intuitively, the minimum wage helps correct the monopsony distortions that depress the employment and wages of lower-earning workers, relative to the case in which only the EITC or a general non-linear tax system is in place.

Minimum Wage and EITC. As discussed, since the EITC increases employment by effectively subsidizing labor supply, firms can afford to pay workers lower before-tax wages, thus appropriating part of the implicit subsidy meant for workers. This issue has led some authors to suggest that a moderate minimum wage may complement transfer programs like the EITC by preventing firms from lowering paid wages (see Neumark and Wascher (2011), Lee and Saez (2012), and Vergara (2022).) Although this effect is present in our model, it is actually relatively muted. The greatest benefit of coupling the EITC with the minimum wage is that the minimum wage helps offset monopsony distortions for workers initially earning wages just below it.

Figure F.2 in Appendix F illustrates the impact of introducing a modest minimum wage of $9.25, once the EITC (budget-equivalent to a $15 minimum wage) is present. We ensure that such a combination of policies leads to a reduction in firm profits equal to the profit loss that would result from just a $15 minimum wage, to make this policy comparable in cost to those considered before. As apparent from the figure, this policy combination leads to a much larger increase in long-run wages and employment for non-college low-wage workers than the EITC alone. One reason is

[34] Estimates of *relative* employment or labor income effects across regions may also be uninformative about the *overall level* effect of a policy in any given region, which could be affected by both local and national factors.

that the added minimum wage prevents firms from lowering before-transfer wages in response to the EITC. Indeed, the blue line in the left panel of Figure F.2 shows that without the minimum wage, firms reduce workers' wages to appropriate part of the subsidy. However, quantitatively, the much more important reason for the greater employment and income response when the two policies are implemented together is that the minimum wage is effective at correcting the monopsony distortions faced by low-wage workers. As illustrated in Figure F.3 in Appendix F, though, too large an increase in the minimum wage (to \$12) may end up hurting these workers for the same reasons highlighted throughout the paper. Hence, our framework supports the argument in the literature that a moderate minimum wage increase can play a valuable role in supporting transfer programs like the EITC but for different reasons than those usually mentioned, namely, for efficiency rather than purely redistributive considerations.

Minimum Wage, EITC, and Progressive Tax and Transfer System. Suppose we introduce in our baseline economy a progressive tax and transfer system as in Heathcote, Storesletten and Violante (2017), who argue that a good approximation to the entire U.S. system results in the after-tax income function $A(w_i) = \varsigma w_i^{1-\tau}$, where ς and τ are parameters that govern the level and progressivity of the system. We assume their same value of 0.181 for the parameter τ, which provides a reasonable description of the overall progressivity of the U.S. system—except at the very bottom of the income distribution, where the system is known to be regressive for those who switch from receiving transfers to working. We then choose the scale parameter ς to ensure that the aggregate net transfer payment is budget-equivalent to a \$15 minimum wage; see Figure F.6.

Briefly, all of our main insights carry over to this case. In particular, the key dynamic trade-offs still emerge. Namely, implementing a large increase in the minimum wage would lead to short- and intermediate-run gains for workers initially earning low wages but at the expense of a long-run decline in their employment; see Figure F.7. Introducing an EITC that is budget-equivalent to a \$10 or \$15 minimum wage still yields short- to intermediate-run benefits that are even larger in the long run. Likewise, an increase in the progressivity of the existing tax and transfer system can help increase the welfare of low-wage workers; see Figure F.7. This result is not surprising given that the introduction of the EITC is one way to increase the progressivity of the system.

9 Conclusion

A fundamental tension within the literature that evaluates labor market policies emerges from the evidence that short-run elasticities of labor-labor substitution are small but the corresponding long-run ones are large. To the best of our knowledge, our paper is the first to attempt to resolve such a puzzle within a quantitative framework. When doing so, a rich set of new insights can be gained into the intertemporal distribution of the benefits and costs of classic redistributive policies. In addressing this puzzle, we have purposely included a minimal set of features that a framework examining such policies needs to integrate: rich heterogeneity within education groups to account for the high degree of within-group wage dispersion; a production structure consistent with the empirical evidence on the elasticities of substitution among capital and labor and among different types of labor in the short and long run; and monopsonisitic competition within a state-of-the art model of frictional labor markets in line with the recent evidence on firm monopsony power.

As such, we hope that researchers will build on this framework to account for any other features that they deem most important in evaluating policies. For instance, one fruitful extension would be to endogenize the distribution of worker skills in response to permanent increases in, say, the minimum wage or the EITC, which we have abstracted from. Intuitively, the demand for low-skilled workers *decreases* in response to a large minimum wage but *increases* in response to an EITC of any size, affecting returns to skills. It is apriori ambiguous, however, if skill acquisition would reinforce or weaken the welfare effects of these policies. To see why, consider a large increase in the minimum wage. On the one hand, since the minimum wage equalizes the wages of workers at the low end of the skill distribution, such a policy may disincentivize them from investing in new skills. On the other hand, since these workers also face a lower probability of finding a job, they may have an incentive to acquire more skills in order to improve their labor market prospects. Exploring the role of skill acquisition requires carefully modeling and disciplining how this process responds to these incentives. Such a model would then have to confront the possibility that workers who can in principle benefit the most from acquiring more skills—those at the bottom of the skill distribution—may also face the lowest returns to doing so. For example, low-wage workers may have lower levels of human capital because they face higher monetary, non-monetary, or opportunity costs to acquiring skills (see, for instance, the work reviewed by French and Taber (2011)).

Another fruitful extension would be to extend our one-final-good framework and consider the pass-through of wage increases due to an increase in the minimum wage, or of wage decreases due to the EITC, to the prices of different final goods. Such price effects have been found to be empirically important (Aaronson and French (2007)). Intuitively, if many of the goods that are disproportionately consumed by low-wage workers, such as fast-food items, are also disproportionately produced by them, then an increase in the minimum wage will tend to raise these goods' prices and further hurt low-wage workers; see Appendix G.2 for a discussion.

Finally, many European countries have a system of wage floors in place contingent on a worker's individual characteristics and labor market experience rather than a statutory national minimum wage as in the United States. Indeed, in Austria, Denmark, Finland, Iceland, Italy, Sweden, Norway, and Germany, multiple wage floors exist, which vary with a worker's age and tenure in firm, occupation, or industry. It could be argued that by their rich design, these systems have some of the flavor of the minimum wage policy that our model implies would be part of an optimal redistributive system. As we have shown in Proposition 2, the wage component of such a policy would imply a minimum wage for each type of worker equal to the competitive (search equilibrium) wage of each type. Hence, these policies are especially relevant for our analysis because type-specific minimum wages can mitigate the incentive for firms to substitute away from low-wage workers. These potential benefits, though, have to be weighted against the potential costs of actual policies that, by not being as finely targeted to worker characteristics that are hard to measure as an optimal policy would be, may even end up further distorting labor markets, by altering the relative prices of the labor services of different workers.[35]

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[35]We have not relied on evidence from these systems in our analysis because many of these countries have also in place other labor market policies, such as employment protection measures, and tax regimes that are quite different from those in the United States. These differences make comparisons difficult, especially in light of the policy interactions discussed in Section 8.

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A From Monopolistic to Monopsonistic Competition

Here we discuss how the upward-sloping labor supply curve for each firm's jobs that our model gives rise to is analogous to the downward-sloping demand curve for each firm's goods that arises in models of monopolistic competition. In these latter models, consumers view each firm's good as imperfectly substitutable with any other. Then, the downward-sloping demand curve of consumers for a firm's good as a function of all firms' prices is the constraint that captures monopoly power in a firm's problem. Analogously, in our setup, workers view each firm's job as imperfectly substitutable with any other. Thus, the upward-sloping supply curve of searchers for a firm's jobs as a function of all firms' wages is the constraint that captures monopsony power in a firm's problem.

To elaborate, recall that standard analyses of monopolistic competition derive the static demand curve for goods of each type j and then impose this demand curve as a constraint on firm j's problem. Equivalently, one could derive the first-order conditions for a consumer's problem and impose the constraint that the marginal utility from buying good j is at least as high as that from buying any other good. Formally, the static part of a consumer's dynamic problem with a standard utility function over differentiated goods, given total expenditure pc, is to choose $\{c_j\}$ to maximize $u(c)$ subject to $\sum_j p_j c_j \leq pc$ with multiplier λ, where $c = (\sum_j c_j^{\frac{\omega-1}{\omega}})^{\frac{\omega}{\omega-1}}$ is the consumption aggregate and p is the associated price index. Then, the first-order condition for good j is

$$u'(c)\left(\frac{c_j}{c}\right)^{-\frac{1}{\omega}} - \lambda p_j = \max_{j'}\left\{u'(c)\left(\frac{c_{j'}}{c}\right)^{-\frac{1}{\omega}} - \lambda p_{j'}\right\}. \tag{24}$$

In a symmetric allocation with $c_{j'} = c$ and $p_{j'} = p$, (24) reduces to

$$u'(c)\left(\frac{c_j}{c}\right)^{-\frac{1}{\omega}} - \lambda p_j \geq u'(c) - \lambda p, \tag{25}$$

which is the participation constraint under monopolistic competition. Notice the similarity of this participation constraint for attracting a consumer to buy from firm j and the participation constraint for attracting a searcher to the labor market (θ_j, w_j) created by firm j, namely, (9). The main difference is that choosing which good to buy given a level of expenditure is a static decision whereas searching for a firm offering a long-term employment contract is a dynamic one.

B Data Description

This appendix contains details about our data sources and targeted moments. We use data from the pooled 2017-2019 ACS. All observations are weighted using the weights provided by the ACS.

Share of College Workers. We define *college* individuals as those individuals who report having a bachelor's degree or higher. During the 2017-2019 period, 31.3% of our sample had at least a bachelor's degree.

Employment Rates. We focus on full-time employment and on workers strongly attached to the labor force. We define individuals as being *full-time* employed if 1) they are currently working at least 30 hours per week; 2) they reported working at least 29 weeks during the prior year; and 3) they reported positive labor earnings during the prior 12 month period. For our 2017-2019 sample, 46.8% of non-college individuals and 62.4% of college individuals worked full-time.

Share of Income Earned by College Workers. For the 2017-2019 period, 37.8% of individuals working full-time were college educated. Conditional on being full-time employed, mean annual earnings for college individuals total \$91,706, whereas mean annual earnings for non-college individuals total \$44,871. Given these statistics, we calculate that 55.5% of all earnings of full-time workers accrued to workers with at least a bachelor's degree.

Table B.1: Average Wages by Education Group in ACS Data

	Less than High School	*High School*	*Some College*	*College*
Average wage	\$16.6	\$19.6	\$21	\$37.4

Note: Average wages of full-time workers by education group in ACS data.

Wage Distributions. We compute hourly wages for our sample of full-time workers by dividing annual labor earning by annual hours worked. We calculate annual hours worked as the product of weeks worked last year and reported usual hours worked. We impose two additional sample restrictions when measuring the wage distribution. First, we restrict the sample to only those workers who report at least \$5,000 of labor earnings during the prior year. Second, we truncate the resulting distribution of hourly wages of each education group at the top and bottom 1%. All wages are converted to 2019 dollars using the June CPI-U. From these data, we compute the median wage as well as the ratios of wages between the 10th percentile and the median and the ratio of wages between the 50th percentile and the median separately for each of the education groups. These moments are used as part of our parameterization strategy. We also show that even though only those three moments are targeted for each education group, our model matches the full distribution of wages for each education group quite closely. As we have emphasized, the heterogeneity of wages within education groups swamps the heterogeneity across education groups, motivating our choice to primarily focus on within-group heterogeneity. Related to this choice, Table B.1 shows that the average wage of *each* education group is higher than \$15 per hour. Hence, modeling within-group heterogeneity is necessary for even a high minimum wage to be binding for any worker.

C Mapping Card and Lemieux (2001) to Our Model

This appendix shows that Card and Lemieux (2001)'s estimated elasticities of substitution across workers, ϕ and ρ, map into the same elasticities in our model despite the fact that Card and Lemieux (2001)'s estimation framework does not allow for monopsony power or search frictions. First, we show that their procedure works arbitrarily well as the size of search frictions becomes small, in which case wage markdowns become constant. Second, we demonstrate that our quantitative model is in practice close to such a limit in two ways. First, we show that Card and Lemieux (2001)'s estimation procedure, when applied to data simulated from our model at their estimated parameter values, recovers the true parameter values almost exactly. Second, we show that when we treat Card and Lemieux (2001)'s estimators of the parameters ϕ and ρ as explicit targets in our moment-matching procedure, we find values very similar to theirs.

C.1 Limit Case with Small Search Frictions

We begin by showing that Card and Lemieux (2001)'s procedure recovers the true parameter values when applied to data generated by our model, as search frictions vanish. The following proposition relates steady-state wage markdowns to search frictions and the degree of monopsony power.

***Proposition* 3.** *Consider the steady state of our model and let i denote a worker type. Then,*

$$\frac{w_i}{\tilde{F}_{ni}} = \frac{1}{1+\frac{1}{\omega}} - (r+\sigma)\frac{\kappa_i}{\lambda_f(\theta_i)}\frac{\omega(1-\eta)-\eta}{(1+\omega)(1-\eta)}\frac{1}{\tilde{F}_{ni}}. \tag{26}$$

.

Proof: Consider the steady-state conditions for vacancy posting and wages:

$$\tilde{F}_{ni} = (r+\sigma)\frac{\kappa_i}{\lambda_f(\theta_i)} + w_i + \frac{1}{\omega}v'(n_i) \tag{27}$$

$$w_i = \eta\left[\tilde{F}_{ni} - \frac{1}{\omega}v'(n_i)\right] + (1-\eta)v'(n_i). \tag{28}$$

Rearrange the wage equation (28) to get an expression for the marginal disutility of work,

$$v'(n_i) = \frac{w_i - \eta\tilde{F}_{ni}}{1-\eta-\frac{\eta}{\omega}}. \tag{29}$$

Plugging this expression into the first-order condition for vacancy posting and collecting terms gives

$$\tilde{F}_{ni}\left[1+\frac{\eta}{\omega(1-\eta-\frac{\eta}{\omega})}\right] = (r+\sigma)\frac{\kappa_i}{\lambda_f(\theta_i)} + w_i\left[1+\frac{1}{\omega(1-\eta-\frac{\eta}{\omega})}\right]. \tag{30}$$

Simplifying this expression yields

$$\tilde{F}_{ni}\frac{\omega}{1+\omega} = (r+\sigma)\frac{\kappa_i}{\lambda_f(\theta_i)}\frac{\omega(1-\eta)-\eta}{(1+\omega)(1-\eta)} + w_i. \tag{31}$$

Finally, dividing by $\tilde{F}_{ni}$ and rearranging terms gives expression (26) in the proposition. ■

Proposition 3 shows that search frictions are the only reason why wage markdowns are not constant in our model: monopsony power in isolation would lead to constant wage markdowns because the disutility of labor supply (1) has a CES form. In our quantitative model, search frictions are small in the sense that the second term in (26) accounts for less than 1% of the overall markdown for the typical worker. Intuitively, the second term depends on the size of search frictions annuitized over the entire life of a match, which is small relative to flow output.

In the limit in which search frictions become arbitrarily small in the sense that the second term in equation (26) converges to zero, Card and Lemieux (2001)'s estimated values for the elasticities of substitution $\widehat{\phi}$ and $\widehat{\rho}$ would exactly equal their true values, that is, $\widehat{\phi} = \phi$ and $\widehat{\rho} = \rho$. To see this, note that in this limit, wages become proportional to marginal products, $w_i = 1/(1+1/\omega)\tilde{F}_{ni}$. Therefore, the ratios of wages across workers would equal the ratios of their marginal products, $w_i/w_j = \tilde{F}_{ni}/\tilde{F}_{nj}$. Since our production function is weakly separable in capital, the ratios of marginal products would fall within the same class considered by Card and Lemieux (2001).

To elaborate, we now turn to show how Card and Lemieux (2001)'s estimators recover the true parameter values in this limiting case of our model. First consider how Card and Lemieux (2001) estimates the within-group elasticity ϕ. Under either their model or in the limit of our model without search frictions, the log ratio of wages of two worker types within an education group is

$$\log\left(\frac{w_i}{w_j}\right) = \log\left(\frac{z_i}{z_j}\right) - \frac{1}{\phi}\log\left(\frac{N_i}{N_j}\right), \tag{32}$$

where N_i and N_j are the measures of workers of two arbitrary types i and j within an education group. Hence, wage ratios are related to the ratio of worker productivities and their relative supply. Card and Lemieux (2001) identifies ϕ from equation (32) by assuming that the relative supply of workers $\log(N_i/N_j)$ changes over time but their relative productivities $\log(z_i/z_j)$ do not. Below, we formalize this variation in our model by assuming that we observe at least two steady states that differ in their initial distributions of types, generating differences in the relative supplies of workers.[36] Taking differences of (32) across these two steady states gives

$$\Delta \log \left(\frac{w_i}{w_j} \right) = -\frac{1}{\phi} \Delta \log \left(\frac{N_i}{N_j} \right). \tag{33}$$

Card and Lemieux (2001) estimates this equation and recovers $\widehat{\phi}$ from the estimated regression coefficient, which corresponds exactly to the true elasticity ϕ. Equation (33) does not exactly hold in our model with search frictions, but we show below that it approximately holds in the sense that running the misspecified regression (33) would recover the true value of ϕ almost exactly.

Now consider how Card and Lemieux (2001) estimates the across-group elasticity ρ. In either their model or in the limit of our model without search frictions, the ratio of wages of workers across educations groups can be written as

$$\log \left(\frac{w_{hi}}{w_{\ell j}} \right) = \log \left(\frac{1-\lambda}{\lambda} \right) + \left(\frac{1}{\phi} - \frac{1}{\rho} \right) \log \left(\frac{\overline{n}_h}{\overline{n}_\ell} \right) + \log \left(\frac{z_{hi}}{z_{\ell j}} \right) - \frac{1}{\phi} \log \left(\frac{N_{hi}}{N_{\ell j}} \right), \tag{34}$$

where $\overline{n}_h$ and $\overline{n}_\ell$ are the aggregated labor inputs defined in (3) and λ is a scale parameter in the production function (so that $(1-\lambda)/\lambda$ captures the degree of "skill bias" in production). In an intermediate step, Card and Lemieux (2001) constructs estimates of z_{hi} and $z_{\ell j}$ using their estimated value for ϕ and the aggregate labor inputs $\overline{n}_h$ and $\overline{n}_\ell$. Then, the only unknowns in equation (34) is the degree of skill bias $(1-\lambda)/\lambda$ and the elasticity of interest ρ. Collecting these unknowns on the right side of (34) and then differencing (34) across steady states gives

$$\underbrace{\Delta \log \left(\frac{w_{hi}}{w_{\ell j}} \right) - \frac{1}{\phi} \Delta \log \left(\frac{\overline{n}_h}{\overline{n}_\ell} \right) + \Delta \frac{1}{\phi} \log \left(\frac{N_{hi}}{N_{\ell j}} \right)}_{\Delta y_i} = \Delta \log \left(\frac{1-\lambda}{\lambda} \right) - \frac{1}{\rho} \Delta \log \left(\frac{\overline{n}_h}{\overline{n}_\ell} \right). \tag{35}$$

Card and Lemieux (2001) assumes that the degree of skill bias $\log[(1-\lambda)/\lambda]$ follows a linear time trend. In this case, its time-difference is constant, so this term becomes a constant in the regression of Δy_i on $-\Delta \log(\overline{n}_h/\overline{n}_\ell)/\rho$. The elasticity ρ can thus be exactly recovered from the regression coefficient on the change in the aggregate labor inputs, $\Delta \log(\overline{n}_h/\overline{n}_\ell)$. Again, equation (33) does not exactly hold in our model, but we show below that it approximately holds.

To summarize, we have shown that in the limit of our model with vanishingly small search frictions, Card and Lemieux (2001)'s estimation strategy exactly recovers the true elasticities of substitution ϕ and ρ. In this case, it is valid for us to use their estimates for the parameter values of our model, despite the fact that our model features monopsony power whereas Card and Lemieux (2001)'s does not. We now turn to showing that this result approximately holds in our quantitative model with search frictions as well.

C.2 Quantitative Results

In order to replicate Card and Lemieux (2001)'s approach in our model, we assume that we observe three steady states corresponding to different distributions of workers and degrees of skill bias in production, which mirrors the empirical variation exploited by Card and Lemieux (2001).

[36] We assume that Card and Lemieux (2001)'s empirical variation corresponds to steady-state changes in our model because their sample covers a forty-year period, by the end of which any transition path would converge to the corresponding steady state in our model.

Specifically, we assume the third and final steady state corresponds to our baseline quantitative model, which is parameterized based on recent data. We then assume that the dispersion of worker abilities z within the two education groups we consider, $\sigma_{\ell z}$ and σ_{hz}, are 10% and 30% lower in the second and first steady states to capture the lower degree of wage dispersion in previous decades. We also assume that the fraction of college-educated workers is 10% and 40% lower in the second and first steady states to reflect the lower rates of educational attainment in past decades. Finally, we assume that the degree of skill bias in production $(1-\lambda)/\lambda$ is 15% and 30% lower in the second and first steady states, which captures the pattern of the college wage premium over time. Although this parameter variation is somewhat arbitrary, we show below that our results are robust to other changes in the distributions of workers or the degree of skill bias.

We start by showing that if we apply Card and Lemieux (2001)'s estimation procedure to data simulated by our model at values for ϕ and ρ equal to their estimates, we essentially recover these values. Thus, their estimating equations effectively apply to our model. To this purpose, we construct Card and Lemieux (2001)'s estimators of the elasticities ϕ and ρ using the procedure outlined above. Specifically, to estimate the within-group elasticity ϕ, we run the regression implied by equation (33) on data simulated from our model as described. We then use the estimated value of $\widehat{\phi}$ to construct estimates of the ratio of type-specific productivities z_i/z_j from equation (32). We next construct the aggregate labor inputs by education group $\overline{n}_h$ and $\overline{n}_\ell$, thus obtaining Δy_i in equation (35). Since Card and Lemieux (2001) assumes that skill-biased technical change follows a linear time trend, the term $\Delta \log[(1-\lambda)/\lambda]$ is constant across steady states. Therefore, to estimate the across-group elasticity of substitution ρ, we double difference (35) to obtain

$$\Delta^2 y_i = -\frac{1}{\rho}\Delta^2 \log\left(\frac{\overline{n}_h}{\overline{n}_\ell}\right). \tag{36}$$

The top panel of Table C.1 shows that Card and Lemieux (2001)'s estimation procedure uncovers the true parameter values nearly exactly: the estimates of $\widehat{\phi} = 4.036$ and $\widehat{\rho} = 1.388$ are both within 1% of the true values $\phi = 4$ and $\rho = 1.4$. Given this similarity, we conclude that the estimation procedure in Card and Lemieux (2001) approximately applies to our model.

We now show that if we treat the estimated values of ϕ and ρ in Card and Lemieux (2001) as estimated parameters of an auxiliary model defined by (33) and (36) as in a standard indirect inference procedure and estimate ϕ and ρ along with all other parameters of our model, we obtain estimates for ϕ and ρ similar to those in Card and Lemieux (2001). The bottom panel of Table C.1 pursues this approach and illustrates that the implied values of ϕ and ρ, which we use in our baseline, are also nearly identical to those in Card and Lemieux (2001).

Finally, Figure C.1 shows that our conclusions in this subsection are robust to alternative ways of generating the time variation required to perform the estimation procedure in Card and Lemieux (2001) on data simulated from our model at their estimated parameter values. Specifically, we construct the Card and Lemieux (2001)'s estimates 500 times, each of which correspond to random draws of the following parameters:

i) dispersion of college ability z, $\sigma_{zh} \sim U[0.6, 0.9]\times$ baseline for initial steady state; second steady state 1/3 between initial and final steady states;

ii) dispersion of non-college ability z, $\sigma_{z\ell} \sim U[0.6, 0.9]\times$ baseline for initial steady state; second steady state 1/3 between initial and final steady states;

iii) fraction of college-educated individuals $\pi_s \sim U[0.6, 0.9]\times$ baseline for initial steady state; second steady state 1/3 between initial and final steady states;

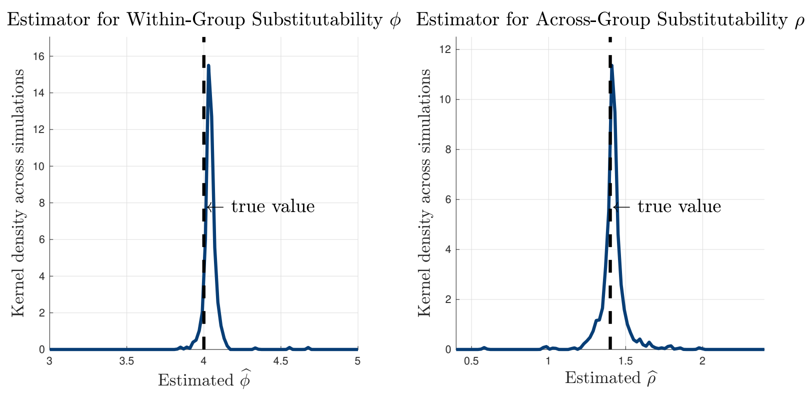
iv) production function parameter $\lambda \sim U[1.1, 1.3]\times$ baseline for initial steady state; second steady state equidistant between initial and final steady states.

Table C.1: Relationship Between Estimates in Card and Lemieux (2001) and Our Estimates

Card and Lemieux (2001)'s Estimates		
	True Value	Estimated Value
Within-group elasticity ϕ	4.00	4.07
Across-group elasticity ρ	1.40	1.45
Targeting Card and Lemieux (2001)'s Estimates		
	Target	Parameter Value
Within-group elasticity ϕ	4.00	4.04
Across-group elasticity ρ	1.40	1.40

Note: Top panel reports Card and Lemieux (2001)'s estimates of the within-group elasticity of substitution ϕ and between-group elasticity ρ (*True Value*) and our estimates of them based on data simulated from our model when the values of ϕ and ρ equal their estimates (*Estimated Value*). Bottom panel reports the recovered values of the elasticities ϕ and ρ if we instead include Card and Lemieux (2001)'s estimators as additional targets in our moment-matching procedure.

Figure C.1: Kernel Densities of Card and Lemieux (2001)'s Estimates on Model-Simulated Data



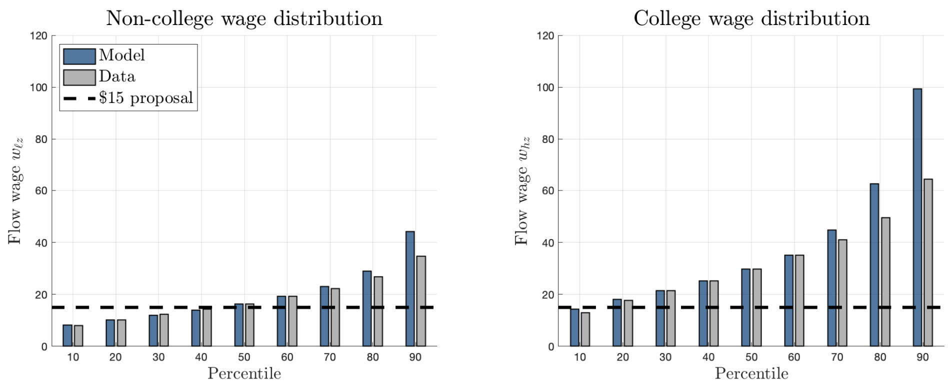
Note: Kernel density of Card and Lemieux (2001)'s estimates across different simulations of our model.

The two panels of Figure C.1 plot the kernel density of the estimates $\widehat{\phi}$ and $\widehat{\rho}$ across these 500 simulations. The distribution of the estimated elasticities $\widehat{\phi}$ is highly concentrated around the true value $\phi = 4$—approximately 99% of the estimates are within 5% of the true value. The distribution of the estimated elasticities $\widehat{\rho}$ is somewhat more dispersed but still concentrated around the true value. Appendix E further shows that our results are robust to the values of ρ in this range.

D Model Fit

We first explore how well our model replicates the observed distributions of wages for workers with and without a college degree. We argue that our model successfully reproduces these features of the data. Figure D.1 compares the wage distributions implied by our model with those from the ACS data. The model well matches the left tail of the wage distribution of each education group by construction, since we target their 50th-10th percentile ratios, which pin down the dispersion parameters of the distributions of worker abilities. As a point of reference, a \$15 minimum wage would bind for approximately 45% of non-college-educated workers in both the model and the data. However, the right tails of the distributions of wages of the two education groups implied by our model are thicker than in the data. As a result, the interquartile range of non-college wages in our model, 2.3, is slightly higher than in the data, 2.1. We are comfortable with this trade-off between the fit of the model at the low and high ends of the two wage distributions because the effects of the minimum wage, the EITC, and the U.S. tax and transfer system that we explore in our quantitative exercises are primarily determined by the left tails of these distributions. Also, across education groups, the model predicts that the median college wage is 1.82 times larger than the median non-college wage, which is in line with a ratio of 1.81 in the data.

FIGURE D.1: Wage Distributions in Model and Data



Note: Wage distributions implied by our model (blue bars) and in the data (grey bars). The wage distribution in the data is measured from the pooled 2017-2019 waves of the ACS, as described in Appendix B.

We now discuss the extent to which our model matches higher-frequency aspects of the data. Specifically, we have also explored the ability of our model to reproduce the evidence on firm-level investment responses to temporary tax incentives. Curtis et al. (2021) estimates the firm-level response of capital and labor to the Bonus Depreciation allowance, a temporary tax incentive for investment, and finds that capital and labor increase roughly proportionally with this incentive. As a further validation of our model, we have replicated the bonus shock as a change in the after-tax relative price of capital and found that capital and labor increase roughly proportionally due to our putty-clay technology. Details are available upon request.

E Additional Results About the Minimum Wage

This appendix contains a number of additional results about the minimum wage.

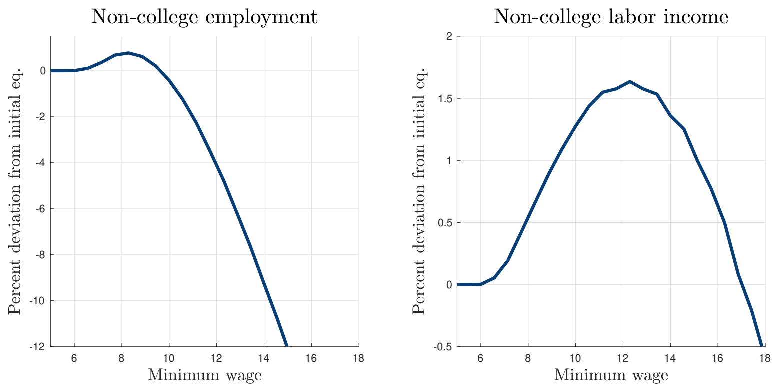
E.1 Long-Run Results

We start by presenting additional long-run results.

Aggregate Laffer Curves. Figure E.1 shows that *aggregate* employment and labor income for non-college workers are also hump-shaped functions of the minimum wage. This shape reflects the aggregation of the individual level responses to the minimum wage illustrated in Figures 2 and 3. Small minimum wage increases—such as increases by $1 or so often studied in the empirical literature—have a small and positive effect on aggregate employment in the long run. A few low-income workers benefit in the long run from such increases and there are no offsetting negative employment effects on workers earning higher wages. Given our parameterization, a minimum wage of around $12 maximizes the labor income of non-college workers as a group.

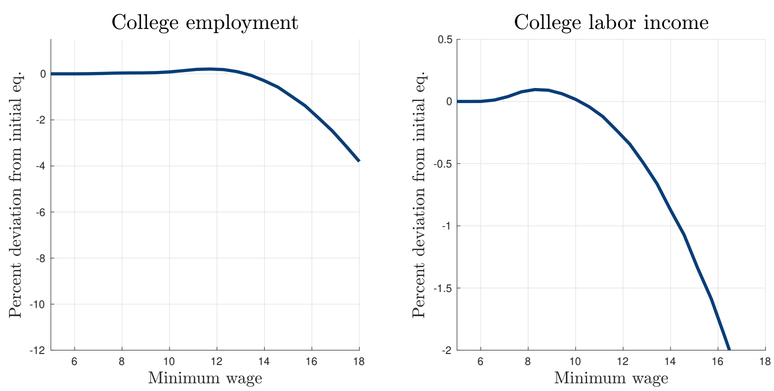
However, as discussed, these aggregate effects mask a large degree of heterogeneity in individual responses. Low-wage workers (such as those initially earning $7.50) are made worse off, whereas higher-wage workers (such as those initially earning $10) are made better off by a large minimum wage increase. As the minimum wage increases, the share of individuals made worse off grows.

Figure E.1: Aggregate Long-Run Minimum Wage Laffer Curves for Non-College Workers



Note: Left panel plots the percentage change in aggregate non-college employment and right panel plots the percentage change in aggregate non-college labor income as a function of the minimum wage.

Figure E.2: Aggregate Long-Run Minimum Wage Laffer Curves for College Workers

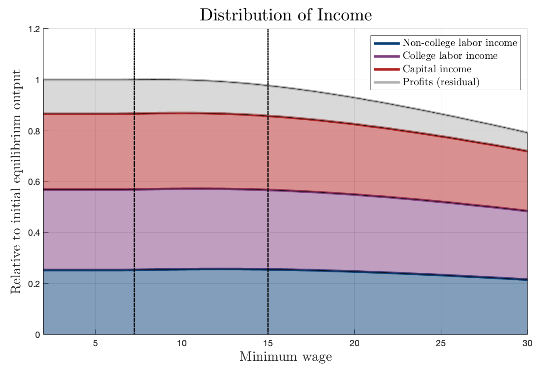


Note: Left panel plots the percentage change in aggregate college employment and right panel plots the percentage change in aggregate college labor income as a function of the minimum wage.

College Workers. The analysis in the main text focuses on non-college workers because the minimum wage primarily binds for them. In fact, relatively few college-educated workers earn less than $15 per hour. However, for sake of completeness, Figure E.2, plots the aggregate employment and income Laffer curves for college workers. The left panel shows that the college employment curve has a shape similar to that for non-college workers, but it starts to decline for higher levels of the minimum wage than for non-college workers. By contrast, the right panel shows that the labor income curve declines by more for college workers than for non-college workers. This occurs because as the minimum wage increases, the correspondingly lower non-college employment and capital stock reduce the marginal product, and therefore the wages, of the college workers.

Composition of Aggregate Income for Different Minimum Wages. In order to illustrate how the minimum wage redistributes aggregate income across different groups, Figure E.3 plots how the minimum wage affects the four components of aggregate income in the long run: non-college labor income, college labor income, capital income, and firms' profits, which, as discussed, primarily reflect firms' monopsony power. Consistent with the labor income Laffer curve in Figure E.1, a \$15 minimum wage raises non-college labor income by 1%. Firm profits fall substantially, suggesting that the minimum wage redistributes resources from firms to workers.

Figure E.3: Distribution of Aggregate Income



Note: Steady-state income shares accounted for by non-college labor income, college labor income, capital income, and firm profits as a function of the minimum wage. The y-axis is normalized so that aggregate income is one without the minimum wage. The x-axis is the level of the minimum that binds on the same fraction of workers as in the data.

This redistribution, however, has a cost: the minimum wage reduces total employment, the aggregate capital stock, and ultimately lowers total output by 2.4%.

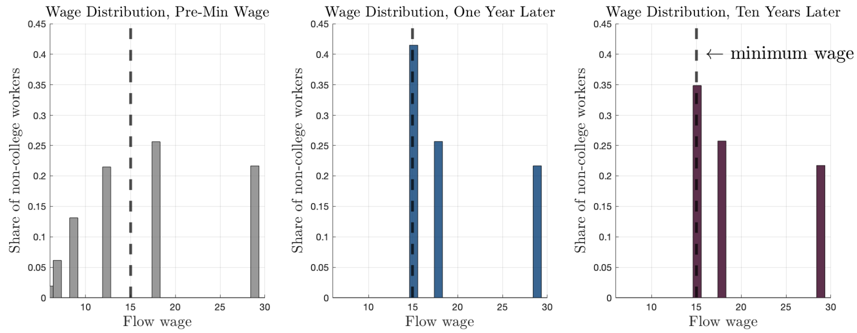
E.2 Transition Dynamics

We provide here additional details about the transition dynamics of our model.

Non-College Wage Distribution Along the Transition. Figure E.4 plots the evolution of the non-college wage distribution following the introduction of a \$15 minimum wage. (This distribution is coarser than the steady state distributions plotted in the main text because we use a coarser grid to compute the transition paths.) The effect on impact (not shown) is that the wages of all workers who initially earn less than the new minimum wage are raised to the new minimum and, hence, are *bunched* there. The effect after one year is similar, although the total mass of workers bunched at the minimum wage is slightly smaller than the original mass of workers earning less than the new minimum wage. The reason is that firms let the original type of capital from the steady state depreciate, invest in new types of capital that are less intensive in the labor services of such workers, and only slowly replace workers in this group with new workers of higher productivity. The right panel shows that this pattern persists, so there is still bunching at the minimum wage over time and the total mass of workers for which it does is now reduced. Hence, our model produces a mass point at the minimum wage, as in Card and Krueger (2015).

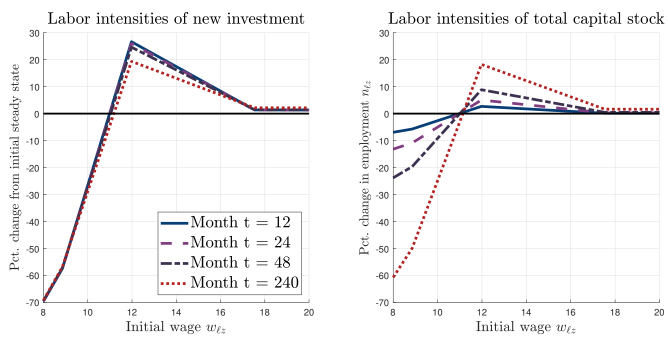
Role of Putty-Clay Capital. In order to understand the role of the putty-clay technology in driving the dynamic responses to policies implied by our model, Figure E.5 plots the labor intensities of new and existing capital along the transition path for non-college workers in response

Figure E.4: Wage Distribution Along the Transition Path



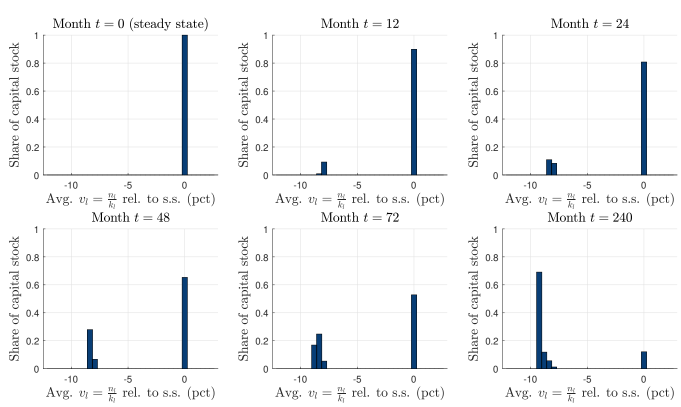
Note: Distribution of wages among non-college workers along the transition path following the introduction of a $15 minimum wage. The left panel shows the steady state wage distribution (along the coarser grid used to compute the transition path). The middle panel plots the wage distribution one year after the introduction of the higher minimum wage. The right panel plots the wage distribution ten years after the introduction of the higher minimum wage.

Figure E.5: Labor Intensities in Response to $15 Minimum Wage at Various Time Horizons



Note: Labor intensities along the transition path following the introduction of a $15 minimum wage, as a function of the initial wage $w_{\ell z}$ of a non-college worker with ability z. Left panel plots non-college labor intensities of new capital $v_{\ell z}$. Right panel plots the corresponding non-college labor intensities of the entire capital stock.

Figure E.6: Distribution of Capital Types Along the Transition Path

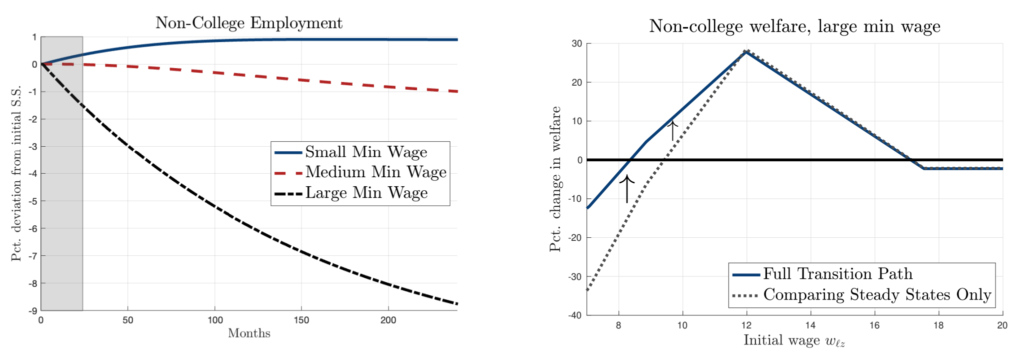


Note: The x-axis in each panel indexes the type of capital by its average non-college labor-to-capital ratio.

to a \$15 minimum wage.[37] The left panel plots the labor-to-capital ratios $v_{\ell z}$ of newly installed capital for each worker type $i = (\ell, z)$ (y-axis) against each worker type's initial wage $w_{\ell z}$ (x-axis). In response to a \$15 minimum wage, firms immediately substitute away from low-ability workers towards higher-ability ones for the newly installed capital. The magnitude of such a substitution is fairly stable at various horizons along the transition path to the new steady state. The right panel of the figure, though, shows that the implied dynamics for the labor intensity of the *total* capital stock is much slower. The reason is that firms continue to operate their existing capital stock along the transition path, which requires the old steady-state mix of worker types chosen before the introduction of the \$15 minimum wage. This old capital accounts for the majority of the aggregate capital stock early on in the transition, but as it depreciates, it is replaced by new capital that is less intensive in the labor services of low-ability workers. Hence, the depreciation rate δ is crucial in determining the speed of transition; see next for an analysis of the robustness of our findings to different values of δ.

Figure E.6 plots the distribution of capital types along the transition path in response to a \$15 minimum wage. Before the introduction of such a minimum, firms hold only one type of capital, namely the type that is optimal at the original steady-state input prices. The higher minimum wage induces firms to invest in less labor-intensive types of capital, but in the early stages of the transition, the new capital accounts for a small share of the total capital stock. Since it takes time for the old capital to depreciate, the bottom right panel shows that, even twenty years after the introduction of the higher minimum wage, more than 10% of the initial stock is still used.

FIGURE E.7: Transition Paths with Lower Capital Depreciation Rate



Note: Transition paths with capital depreciation rate δ equal to 10% annually. Left panel plots the percentage change in non-college employment after the introduction of a \$15 minimum wage. Right panel plots the resulting change in welfare (blue line) and the change in welfare implied by only comparing steady states (black line).

Role of Capital Depreciation Rate. Figure E.7 plots the transition paths following the introduction of a \$15 minimum wage under an alternative parameterization with a capital depreciation rate of $\delta = 10\%$ annually. This depreciation rate is lower than the 15% annual depreciation rate in our baseline because it incorporates structures among capital, in addition to equipment and software. The left panel shows that with this lower rate of depreciation, the transition dynamics for employment are more protracted than in the main text. This result occurs because it takes longer for firms to re-configure their production away from low-wage workers. Accordingly, the long-run costs of the minimum wage for these workers takes longer to materialize. The right panel shows that the implied welfare loss for low-wage workers is smaller when $\delta = 10\%$ than when $\delta = 15\%$.

[37] The curves are less smooth along the transition path than in steady state because we use a coarser grid of z-types when computing the transition paths.

Hence, our main findings about the role of putty-clay frictions in governing the time profile of the impact of a \$15 minimum wage are even starker with a 10% depreciation rate.

E.3 Results with Capital-Skill Complementarity

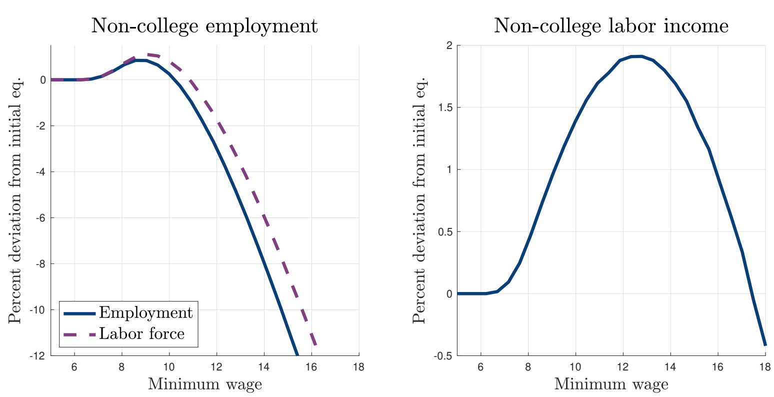
So far, we have assumed that capital is equally substitutable with college and non-college labor (see equation (2)). In influential work on the determinants of the wage skill premium, Krusell et al. (2000) argue, instead, that capital is more substitutable with non-college labor than it is with college labor—a configuration they refer to as *capital-skill complementarity*. Here, we show that our main results are robust to this alternative specification of the production structure. This dimension of robustness of our results is due to the distributional impact of the policies we consider being primarily determined by the substitutability of workers within an education group rather than by their substitutability across education groups or with capital.

TABLE E.1: Targeted Moments in Baseline vs. Capital-Skill Complementarity

Moment	Description	Data	Baseline Model	KORV Model
Average wage markdown				
$\mathbb{E}[w_{ni}]/\mathbb{E}[\tilde{F}_{ni}]$	Average wage markdown	0.75	0.76	0.75
Wage distributions				
$w_{\ell 50}/w_{\ell 10}$	Non-college 50th-10th ratio	2.04	1.98	1.90
w_{h50}/w_{h10}	College 50th-10th ratio	2.30	2.07	2.06
Income shares				
$\mathbb{E}[w_i n_i]/Y$	Aggregate income share	57%	57%	57%
$\pi_h \mathbb{E}[w_{hz} n_{hz}]/\mathbb{E}[w_i n_i]$	College income share	55%	55%	55%
Employment rates				
$\mathbb{E}_\ell[n_i]$	Non-college employment rate	47%	47%	47%
$\mathbb{E}_h[n_i]$	College employment rate	62%	61%	61%
Unemployment rate				
$\mathbb{E}[s_i]/(\mathbb{E}[s_i] + \mathbb{E}[n_i])$	Average unemployment rate	5.9%	5.9%	5.9%
Job-finding rate				
$E[\lambda_w(\theta_{ijt})]$	Average job-finding rate	45%	44%	44%

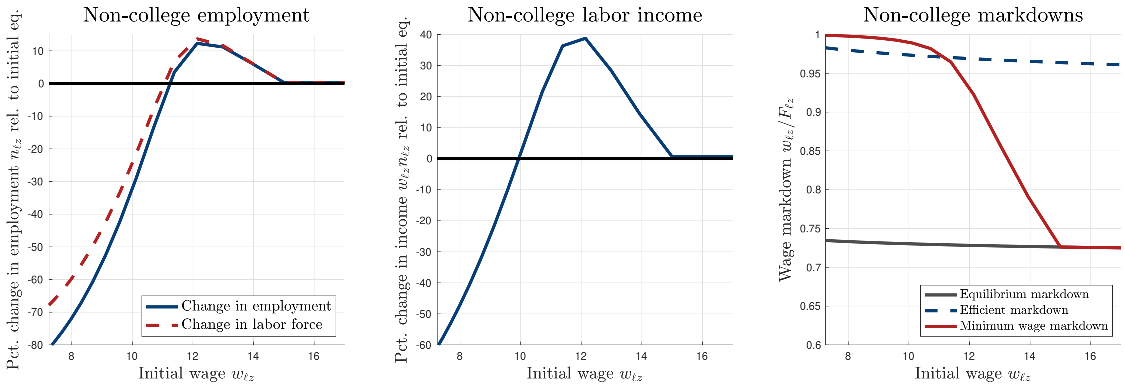
Note: *Baseline Model* refers to our baseline. *KORV model* refers to the model with capital-skill complementarity.

FIGURE E.8: Aggregate Long-Run Laffer Curves with Capital-Skill Complementarity



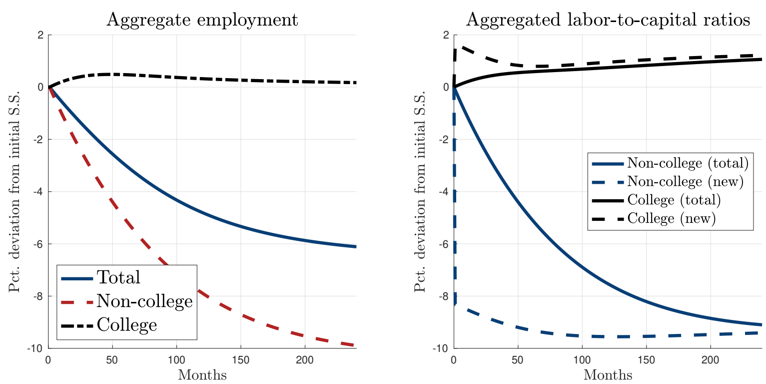
Note: Left panel plots the percentage change in aggregate non-college employment and participation and right panel plots the percentage change in aggregate non-college labor income as a function of the minimum wage.

FIGURE E.9: Distributional Effects of \$15 Minimum Wage with Capital-Skill Complementarity



Note: Left panel plots the percentage change in employment and participation, middle panel plots the percentage change in labor income, and right panel plots the levels of the wage markdown $w_i/\tilde{F}_{ni}$ for three different parameterizations: *i)* $\underline{w} = 0$ (*Equilibrium markdown*); *ii)* $\underline{w} = 0$ and $\omega \to \infty$ (*Efficient markdown*); and *iii)* $\underline{w} = \$15$ (*Minimum wage markdown*). The x-axis corresponds to the wage $w_{\ell z}$ of a type-z non-college worker in the initial steady state.

FIGURE E.10: Employment Dynamics Along the Transition with Capital-Skill Complementarity



Note: Transition paths following the introduction of a \$15 minimum wage in the model with capital-skill complementarity, starting from the initial steady state. Left panel plots the employment of non-college workers, college workers, and total employment. Right panel plots the dynamics of labor-to-capital ratios.

In this alternative version of our model, the long-run production structure in (2) is replaced by

$$F(k_{jt}, \bar{n}_{\ell jt}, \bar{n}_{hjt}) = \left[\psi(\bar{n}_{\ell jt})^{\frac{\rho-1}{\rho}} + (1-\psi)G(k_{jt}, \bar{n}_{hjt})^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}, \tag{37}$$

$$G(k_{jt}, \bar{n}_{hjt}) = \left[\lambda\,(k_{jt})^{\frac{\alpha-1}{\alpha}} + (1-\lambda)(\bar{n}_{hjt})^{\frac{\alpha-1}{\alpha}}\right]^{\frac{\alpha}{\alpha-1}}. \tag{38}$$

Now, the parameter α measures the elasticity of substitution between capital and college labor, whereas the parameter ρ measures the elasticity of substitution between non-college labor and the capital-college labor bundle $G(k_{jt}, \bar{n}_{hjt})$. Krusell et al. (2000) estimate that $\rho > \alpha$, which implies that non-college labor is more substitutable with capital than is college labor. The aggregate labor inputs $\bar{n}_{\ell jt}$ and $\bar{n}_{\ell jt}$ depend on workers' type-specific ability or productivity z_i, whose substitutability among workers within each education group is determined by the elasticity parameter ϕ as in the main text. We reparameterize the model with this alternative production structure by targeting the same statistics as in our baseline parameterization. Table E.1 shows that this alternative model matches the targeted moments about as well as our baseline model.

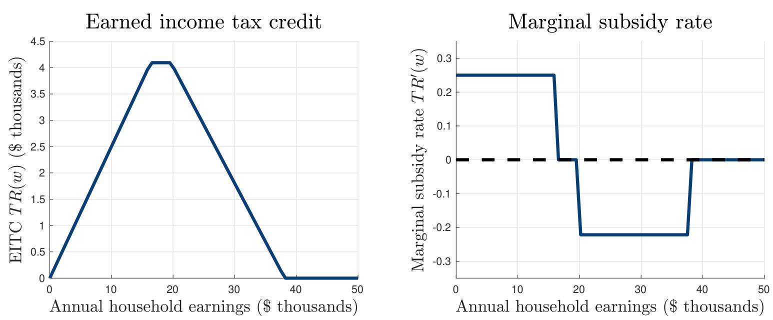
The long-run effects of the minimum wage in this alternative model are also very similar to those in our baseline model. Figure E.8 plots the employment and labor income Laffer Curves for non-college workers—they are nearly identical to those in Figure E.1. For example, a $15 minimum wage decreases non-college employment by about 12% in both cases. Figure E.9 shows that the distributional impact of a $15 minimum wage across non-college workers of different abilities is also nearly identical to its counterpart illustrated in Figure 3 in the main text.

Figure E.10 additionally shows that the transition path in this alternative model is very similar to that in the baseline model. As in our baseline model, it takes employment more than twenty years to converge to its new steady state level due to the putty-clay frictions.

F Additional Results About Alternative Policies

This appendix contains additional results pertaining to the minimum wage, the EITC, the general tax and transfer system, and their interplay. Figure F.1 plots the average and marginal subsidy rates implied by the EITC policy discussed in Section 5.

Figure F.1: EITC Schedule

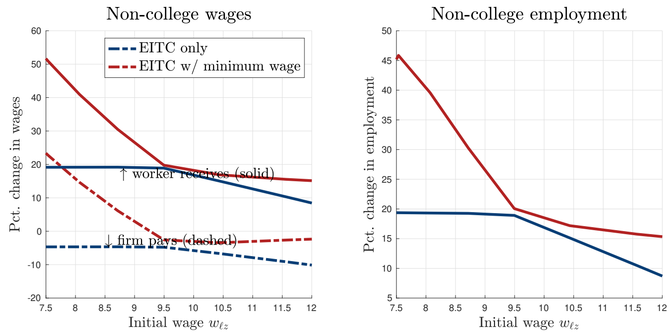


Note: Left panel plots an EITC schedule that is budget-equivalent to a $15 minimum wage. Right panel plots the implied marginal subsidy rate. In each panel, the x-axis rescales steady-state labor income to annual labor income assuming each household works 1800 hours per year.

Complementarity Between EITC and the Minimum Wage. Figure F.2 shows that combining the EITC (budget-equivalent to a $15 minimum wage) with a minimum wage of $9.25 leads to a larger increase in the employment and labor income of non-college workers than under the EITC in isolation. Intuitively, as discussed in the main text, integrating the two policies is beneficial for low-wage non-college workers because the minimum wage directly alleviates the monopsony distortions faced by them. It also prevents firms from lowering the before-transfer wages they pay to those workers and thereby from appropriating some of the benefits of the EITC. However, Figure F.3 shows that combining the EITC with a $12 minimum wage leads to worse outcomes for the lowest-wage workers than under the EITC in isolation. This higher minimum is above the efficient level of wages for these workers and so lowers their employment.

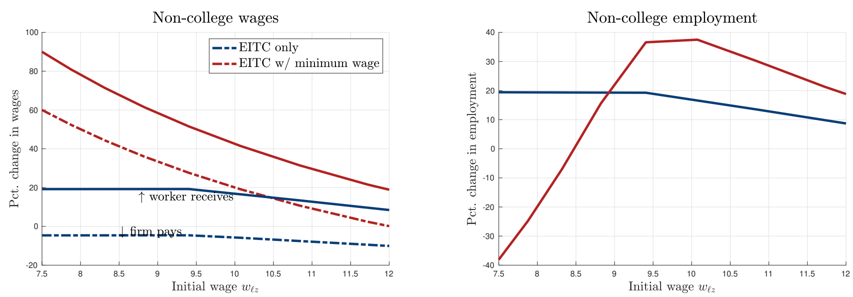
Small EITC. We now show that the effects of the EITC are qualitatively similar across versions of the policy of different sizes. Specifically, suppose that we introduce in our baseline economy an EITC that is budget-equivalent to a $10 minimum wage and is therefore smaller than the EITC examined in the main text. Figure F.4 shows that in the long run, the impact of this smaller EITC on the employment and labor income of non-college workers is similar to that of the larger EITC,

Figure F.2: Long-Run Effects of EITC with and without \$9.25 Minimum Wage



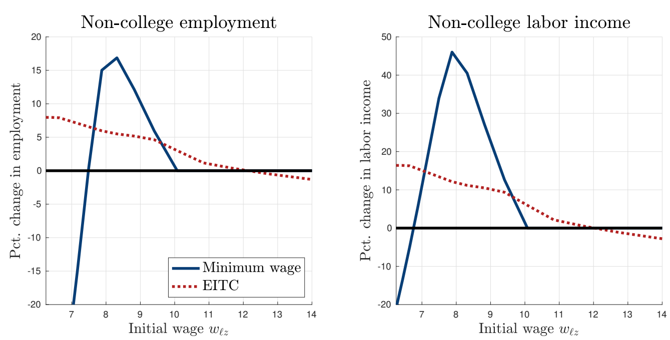
Note: Steady-state wages (left panel) and employment (right panel) of select non-college z-types under the EITC and the EITC combined with a \$9.25 minimum wage. The y-axis is the percentage change relative to the initial steady state and the x-axis is the wage $w_{\ell z}$ of a z-type worker in the initial steady state. Dashed lines denote the wages that firms offer, as opposed to those that workers receive in the two cases.

Figure F.3: Long-Run Effects of EITC with and without \$12 Minimum Wage



Note: Steady-state wages (left panel) and employment (right panel) of select non-college z-types under the EITC and the EITC combined with a \$12 minimum wage. The y-axis is the percentage change relative to the initial steady state and the x-axis is the wage $w_{\ell z}$ of a z-type worker in the initial steady state. Dashed lines denote the wages that firms offer, as opposed to those that workers receive in the two cases.

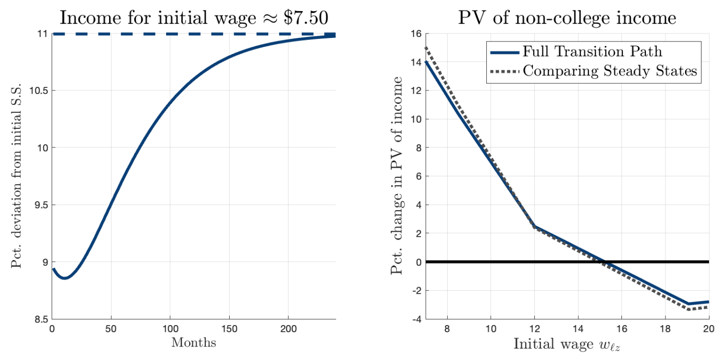
Figure F.4: Long-Run Effects of Small EITC



Note: Percentage change in steady-state employment (left panel) and labor income (right panel) of non-college workers under an EITC that is budget-equivalent to a \$10 minimum wage, relative to the initial steady state. The x-axis is the initial wage $w_{\ell z}$ of a z-type worker. *Minimum wage* plots the effects of a \$10 minimum wage.

although the increase in employment is naturally smaller. Figure F.5 shows that these benefits of the EITC to low-wage workers take time to fully materialize due to our putty-clay frictions.

Figure F.5: Dynamic Effects of Small EITC



Note: Dynamic effects of an EITC budget-equivalent to a $10 minimum wage. Left panel plots labor income for non-college workers initially earning the lowest wages. Right panel plots the change in the present value of income among non-college workers. The blue line computes the present value of labor income over the entire transition path and the black line only compares steady states. The x-axis is the initial wage $w_{\ell z}$ of a z-type worker.

Progressive Tax and Transfer System. Figure F.6 plots the system described in Section 8. Recall that this system is not meant to provie an accurate description of the entire tax and transfer system in the United States, which raises substantial revenues from households. In fact, once we introduce the system described in our model, on net we transfer revenues to households—the size of the implied transfer is equivalent to the loss in profits from a $15 minimum wage—but in a way that mirrors the overall progressivity of the U.S. tax code. In particular, households at the bottom of the income distribution receive substantial transfers, which are phased-out as income rises. Households at the top of the income distribution, instead, pay positive taxes.

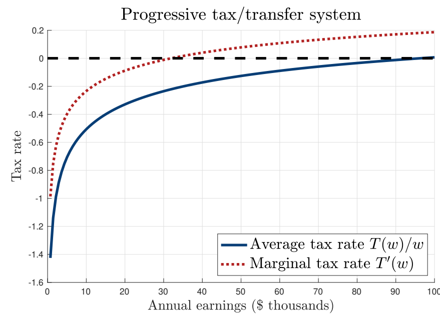
The blue lines in Figure F.7 show the long-run effects of this progressive tax and transfer transfer system. Analogously to the EITC, this system increases the employment and income of non-college low-wage workers. In fact, given that the system raises additional revenues from the top earners in the economy, it is able to transfer more resources to low-wage workers and therefore has a larger beneficial effect than the EITC. The red lines in the figure show the effect of an even more progressive tax and transfer system with $\tau = 0.463$, which is meant to capture the degree of progressivity of the tax and transfer system in Denmark.[38] This more progressive system further increases the employment of low-wage non-college workers but decreases that of college workers more than the U.S. system does (not pictured), which underscores the distributional trade-offs associated with different levels of progressivity.

G Additional Robustness and Discussion

Here, we examine the role of specific parameters in determining the long-run effects of the minimum wage and the EITC. We also discuss the potential effects of these policies on output prices.

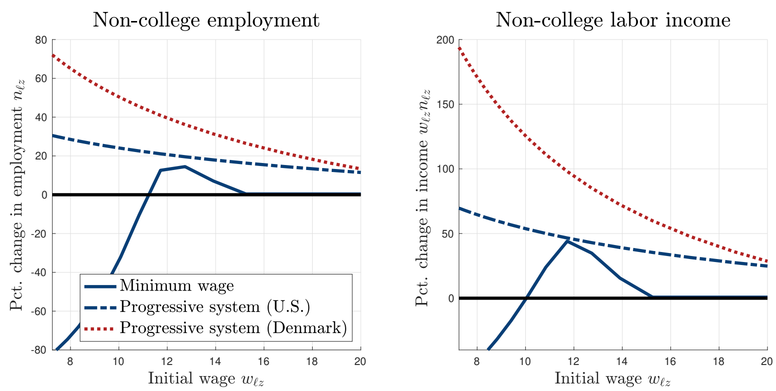
[38]Heathcote, Storesletten and Violante (2020) estimate the progressivity of the tax system in Denmark and other countries. Due to data limitations, though, these estimates only include taxes, thus excluding transfers. We impute a value for the progressivity of the Danish tax and transfer system by scaling our baseline progressivity parameter for the U.S. system by the ratio of the progressivity of the Danish tax system to the U.S. tax one (ignoring transfers).

Figure F.6: Progressive Tax and Transfer System



Note: Average tax rates $T(w)/w$ and marginal tax rates $T'(w)$ implied by the budget-equivalent tax and transfer system described in the main text with the U.S. level of progressivity $\tau = 0.181$. In each panel, the x-axis rescales steady-state labor income to annual labor income assuming each household works 1800 hours per year.

Figure F.7: Effect of Progressive Tax and Transfer System on Non-College Workers



Note: Steady-state employment (left panel) and labor income (right panel) for three policies: a $15 minimum wage (solid line), a budget-equivalent progressive tax and transfer system with the U.S. level of $\tau = 0.181$ (dash-dotted line), and a budget-equivalent tax and transfer system with the Danish level of $\tau = 0.463$ (dotted line). The y-axis is the percentage change relative to the initial steady state and the x-axis is the initial wage $w_{\ell z}$ of a z-type worker.

G.1 Additional Robustness

Table G.1 shows the robustness of our results about the long-run effect of a $15 minimum wage and a budget-equivalent EITC to model parameters not explicitly discussed in the main text: the production elasticities ρ and α, the value of the labor supply elasticity as governed by the parameter γ_n, and the degree of search frictions as governed by the parameters κ_0, η, and σ.

First, consider the production elasticities. We set a higher value of the between-education-group elasticity $\rho = 4$ from recent work by Bils, Kaymak and Wu (2020), as opposed to $\rho = 1.4$ in our baseline, and find that our main results are not much sensitive to even large changes in the value of this parameter.[39] We then consider two different values of the elasticity of substitution between capital and labor α, corresponding to greater complementarity than Cobb-Douglas ($\alpha = 0.7$ from Oberfield and Raval (2021)) and greater substitutability than Cobb-Douglas ($\alpha = 1.25$ from

[39]The findings in Bowlus et al. (2021) suggest an even greater substitutability between these workers than reported in previous studies. They document an elasticity of substitution between high-school- and college-educated workers that ranges between 3 and 8, depending on the years considered and the modelling of skill-biased technical change.

Karabarbounis and Neiman (2014)). In either case, results are fairly similar to those from our baseline. Overall, these results underscore that the most important margin of adjustment on the part of firms in response to any policy concerns the within-education-group substitutability among workers highlighted above, as opposed to either the between-education-group substitutability among them or the substitutability between capital and labor.

TABLE G.1: Robustness of Long-Run Results for a $15 Minimum Wage

	\$15 Minimum Wage		Budget-equivalent EITC	
Parameterization	Δn_ℓ	$\Delta w_\ell n_\ell$	Δn_ℓ	$\Delta w_\ell n_\ell$
Baseline	-12.1%	1.1%	5.2%	3.8%
Other production elasticities				
Cross-group $\rho = 4$ (Bils, Kaymak and Wu, 2020)	-12.6%	-0.1%	5.8%	4.7%
Capital-labor $\alpha = 0.7$ (Oberfield and Raval, 2021)	-9.7%	2.4%	4.7%	3.5%
Capital-labor $\alpha = 1.25$ (Karabarbounis and Neiman, 2014)	-12.3%	1.3%	5.1%	3.6%
Other labor supply elasticity				
Labor supply elasticity $\gamma_n = 1/2$ (Chetty et al., 2011)	-14.2%	0.2%	3.0%	3.4%
Other search frictions (sensitivity to 50% above baseline)				
Vacancy posting costs κ_0	-12.1%	1.1%	5.2%	3.8%
Matching function elasticity η	-11.7%	1.1%	4.8%	3.5%
Job separation rate σ	-12.4%	0.9%	5.2%	4.0%

Note: Change in steady-state outcomes after a $15 minimum wage. Δn_ℓ is the change in non-college employment and $\Delta w_\ell n_\ell$ is the change in non-college labor income. *Baseline* refers to our baseline model. *Other production elasticities* refers to a re-parameterized model with different values for the production elasticities. *Other labor supply elasticity* refers to a re-parameterized model with a different value of γ_n. *Other search frictions* refers to a re-parameterized model with each search parameter set to 50% above its baseline value.

It turns out that our results do not vary much either when we set the parameter γ_n for the elasticity of labor supply equal to 0.5, which is a value representative of the values in the *micro range* from Chetty et al. (2011), as opposed to our benchmark value of 1 from their *macro range.*

The bottom rows of Table G.1 show that different values of the parameters that govern search frictions have minimal impact on our results. Since there is limited external evidence on the magnitude of these parameters, we simply perform a sensitivity analysis in which we increase their size by 50%. Our results do not change much with these alternative parameterizations.

G.2 Discussion: Price Effects

Throughout our analysis, we have assumed that there is only one final good and so abstracted from the potential pass-through of wage increases, say, due to an increase in the minimum wage, or decreases, say, due to the EITC, to the prices of different final goods, which may naturally arise in a multi-good economy. Consider first the price effects in response to, say, an increase in the minimum wage when there is a bundle of final goods. Intuitively, an increase in the minimum wage may have an indirect negative effect on relatively poorer workers. For instance, fast-food prices may increase in response to a minimum wage increase, given that many low-wage workers are employed in this sector. Now, expenditure on fast food accounts for a larger budget share for low-wage workers than for higher-wage workers. As a result, the price effects of an increase in the minimum wage may dampen the welfare benefits (or amplify the welfare losses) for low-wage workers, since such workers would now be disproportionally facing higher prices.

Although we leave formally modeling these potential price effects to future work given its complexity—we would need to augment our already rich model with a demand system for heterogeneous goods—two comments are worth making. First, if price adjustments occur over time due to some form of price stickiness, then our qualitative findings about the impact of policies in the short and intermediate run are likely to hold. In particular, the minimum wage may simply have more perverse effects in the long run. But if close substitutes exist for the goods produced by industries whose workers are especially exposed to a minimum wage increase, then this pass-through of wages to prices may be limited even in the long run. Namely, if fast food were to become more expensive, then consumers of it may opt for cheaper varieties of, say, ready-to-eat frozen food, thus curbing fast-food firms' incentives to raise prices.

Second, to the extent that the same workers disproportionally facing price increases are those whose wages also increase due to an increase in the minimum wage, then these workers would experience approximately similar effects to those discussed so far, as long as the pass-through from wages to prices is less than one-to-one. Naturally, workers facing price increases who are priced out of the labor market would experience even more adverse welfare effects.

Hence, the key distributional trade-offs we have emphasized so far are likely to survive to this richer framework. The logic behind the potential price effects of the EITC is analogous, although of the opposite sign.

Online Appendix: Equilibrium Characterization

(*current version at* `https://drive.google.com/file/d/1Y4qowGOfV7Uw3SfuL3Cj0jfJj7Fwb2R3/view`)

This appendix provide details about the characterization of the equilibrium of our model. Section A describes the household's problem, which shows how our model generalizes Robinson (1933)'s firm-specific labor supply curve to a dynamic model with search frictions and long-term contracts (the *participation constraint* in the main text). Section B describes the firm's problem with standard capital, which shows how we simplify firms' long-term employment problem by collecting Lagrange multipliers following Marcet and Marimon (2019). Finally, Section C describes the firm's problem with putty-clay capital, which shows how we aggregate across capital types to simplify its solution.

A Household Problem

The household problem is the same whether or not there is a minimum wage and whether capital is standard or putty-clay.

Utility Maximization Problem. Each type-i family maximizes $\sum_{t=0}^{\infty} \beta^t u(c_{it}, n_{it}, s_{it})$ subject to

$$\beta^{t+1}\mu_{ijt+1} : n_{ijt+1} \leq (1-\sigma)n_{ijt} + \lambda_w(\theta_{ijt})s_{ijt} \tag{39}$$

$$\gamma_i : \textstyle\sum_{t=0}^{\infty} Q_{0,t}c_{it} \leq \sum_{t=0}^{\infty} Q_{0,t} \sum_j \lambda_w(\theta_{ijt-1})s_{ijt-1}W_{ijt} + \psi_i\Pi_0. \tag{40}$$

Solving the Household Problem. The first-order conditions (FOCs) for consumption at t, employment at firm j at $t+1$, and searching for firm j at t are

$$c_{it} : \beta^t u_{cit} = \gamma_i Q_{0,t} \quad \text{so } \beta\frac{u_{cit+1}}{u_{cit}} = Q_{t,t+1} \tag{41}$$

$$n_{ijt+1} : \beta^{t+1}\mu_{ijt+1} = \beta^{t+1}u_{nit+1}\left(\frac{n_{ijt+1}}{n_{it+1}}\right)^{\frac{1}{\omega}} + \beta^{t+2}\mu_{ijt+2}(1-\sigma) \tag{42}$$

$$s_{ijt} : -\beta^t u_{sit} = \beta^{t+1}\mu_{ijt+1}\lambda_w(\theta_{ijt}) + \gamma_i Q_{0,t+1}\lambda_w(\theta_{ijt})W_{ijt+1}, \tag{43}$$

where in the employment FOC (42), we used the following property evaluated at $t+1$ instead of t

$$\begin{aligned}\frac{\partial u_{it}}{\partial n_{ijt}} &= u_{nit}\frac{\partial n_{it}}{\partial n_{jit}} = u_{nit}\frac{\partial\left(\sum_j n_{ijt}^{\frac{1+\omega}{\omega}}\right)^{\frac{\omega}{1+\omega}}}{\partial n_{ijt}} = u_{nit}\left(\frac{\omega}{1+\omega}\right)\left(\textstyle\sum_j n_{ijt}^{\frac{1+\omega}{\omega}}\right)^{\frac{\omega}{1+\omega}-1}\left(\frac{1+\omega}{\omega}\right)n_{ijt}^{\frac{1+\omega}{\omega}-1} \\ &= u_{nit}\left(n_{it}^{\frac{1+\omega}{\omega}}\right)^{-\frac{1}{1+\omega}}n_{ijt}^{\frac{1}{\omega}} = u_{nit}n_{it}^{-\frac{1}{\omega}}n_{ijt}^{\frac{1}{\omega}} = u_{nit}\left(\frac{n_{ijt}}{n_{it}}\right)^{\frac{1}{\omega}}.\end{aligned} \tag{44}$$

We now use the consumption FOC (41) to substitute out the Lagrange multiplier γ_i on the budget constraint and then we manipulate the labor FOC (42) and the search FOC (43) to obtain a more convenient form. We rewrite the labor FOC (42) as

$$\mu_{ijt+1} = u_{nit+1}\left(\frac{n_{ijt+1}}{n_{it+1}}\right)^{\frac{1}{\omega}} + \beta(1-\sigma)\mu_{ijt+2}.$$

Divide this equation by u_{cit+1} to obtain

$$\frac{\mu_{ijt+1}}{u_{cit+1}} = \frac{u_{nit+1}}{u_{cit+1}}\left(\frac{n_{ijt+1}}{n_{it+1}}\right)^{\frac{1}{\omega}} + \beta(1-\sigma)\frac{\mu_{ijt+2}}{u_{cit+2}}\frac{u_{cit+2}}{u_{cit+1}},$$

then define $V_{ijt+1} \equiv \mu_{ijt+1}/u_{cit+1}$ and substitute it to obtain a recursive version of V_{ijt+1}

$$V_{ijt+1} = \frac{u_{nit+1}}{u_{cit+1}}\left(\frac{n_{ijt+1}}{n_{it+1}}\right)^{\frac{1}{\omega}} + (1-\sigma)\beta\frac{u_{cit+2}}{u_{cit+1}}V_{ijt+2}. \tag{45}$$

Next, note that for each t the consumer's FOC implies $Q_{t+1,t+2} = \beta u_{cit+2}/u_{cit+1}$, so (45) becomes

$$V_{ijt+1} = \frac{u_{nit+1}}{u_{cit+1}}\left(\frac{n_{ijt+1}}{n_{it+1}}\right)^{\frac{1}{\omega}} + (1-\sigma)Q_{t+1,t+2}V_{ijt+2}, \tag{46}$$

which, evaluated at t, is

$$V_{ijt} = \frac{u_{nit}}{u_{cit}}\left(\frac{n_{ijt}}{n_{it}}\right)^{\frac{1}{\omega}} + (1-\sigma)Q_{t,t+1}V_{ijt+1}. \tag{47}$$

Next, use $Q_{0,t} = Q_{0,1}Q_{1,2}\ldots Q_{t-1,t}$ and note that the solved-out version of V_{ijt+1} implies that the present value of the marginal disutility of working starting in period t in consumption units is

$$V_{ijt} = \textstyle\sum_{s=0}^{\infty} Q_{t,t+s}(1-\sigma)^s \frac{u_{nit+s}}{u_{cit+s}}\left(\frac{n_{ijt+s}}{n_{it+s}}\right)^{\frac{1}{\omega}}; \tag{48}$$

V_{ijt} is the marginal disutility per family and $\mu_i V_{ijt}$ is the marginal disutility of all type-i families.

Consider next the search FOC (43). Substitute $\gamma_i Q_{0,t+1} = \beta^{t+1}u_{cit+1}$ and rearrange to obtain

$$s_{ijt}: -\beta^t u_{sit} = \beta^{t+1}\mu_{ijt+1}\lambda_w(\theta_{ijt}) + \beta^{t+1}u_{cit+1}\lambda_w(\theta_{ijt})W_{ijt+1}.$$

Now divide by $\beta^t u_{cit}$ to get

$$s_{ijt}: -\frac{u_{sit}}{u_{cit}} = \beta\lambda_w(\theta_{ijt})\frac{u_{cit+1}}{u_{cit}}\frac{\mu_{ijt+1}}{u_{cit+1}} + \beta\frac{u_{cit+1}}{u_{cit}}\lambda_w(\theta_{ijt})W_{ijt+1}.$$

Finally, plug in $\frac{\mu_{ijt+1}}{u_{cit+1}} = V_{ijt+1}$ and use $Q_{t,t+1} = \beta u_{cit+1}/u_{cit}$ to obtain

$$s_{ijt}: -\frac{u_{sit}}{u_{cit}} = Q_{t,t+1}\lambda_w(\theta_{ijt})\left(W_{ijt+1} + V_{ijt+1}\right) \text{ for all } j. \tag{49}$$

This equation (49) implies the *participation constraint* from the main text,

$$\lambda_w(\theta_{ijt})\left(W_{ijt+1} + V_{ijt+1}\right) \geq \lambda_w(\theta_{it})\left(W_{it+1} + V_{it+1}\right) = \mathcal{W}_{it}. \tag{50}$$

Since families can perfectly insure the idiosyncratic risk of their members and there are no aggregate shocks, it is without loss to adopt the convention that a firm fulfills its present-value wage offer W_{ijt} by offering a constant period wage w_{ijt} over the course of a match that begins at t so that

$$W_{ijt} = w_{ijt} + (1-\sigma)Q_{t,t+1}w_{ijt} + (1-\sigma)^2 Q_{t,t+2}w_{ijt} + \ldots, \tag{51}$$

where $Q_{t,s}$ is the price of goods in $s > t$ in units of goods in t.

Symmetric Steady State. In a steady state in which all firms j choose the same policies for workers of type i these conditions become

$$n_i : V_i = \frac{u_{nit}}{u_{cit}} + \beta(1-\sigma)V_i \text{ so } V_i = \frac{u_{ni}/u_{ci}}{1-\beta(1-\sigma)} \tag{52}$$

and

$$s_i : -\frac{u_{si}}{u_{ci}} = \beta\lambda_w(\theta_i)\left(W_i + V_i\right) = \frac{\beta}{1-\beta(1-\sigma)}\lambda_w(\theta_i)\left(w_i + u_{ni}/u_{ci}\right). \tag{53}$$

To understand the second equality in (53) above, note that in a steady state $Q_{t,t+k} = \beta^k$ so that the present value of wages in (51) reduces to

$$W_i = w_i + \beta(1-\sigma)w_i + \beta^2(1-\sigma)^2 w_i + \dots \text{ so } W_i = \frac{w_i}{1-\beta(1-\sigma)}. \tag{54}$$

With GHH preferences $u(c_i, s_i, n_i) = U(c_i - h(s_i) - v(n_i))$, we have

$$\frac{u_{ni}}{u_{ci}} = -v'(n_i) \text{ and } \frac{u_{si}}{u_{ci}} = -h'(s_i)$$

so (52) and (53) become

$$V_i = -\frac{v'(n_i)}{1-\beta(1-\sigma)} \text{ and } h'(s) = \frac{\beta}{1-\beta(1-\sigma)}\lambda_w(\theta_i)\left[w_i - v'(n_i)\right] \tag{55}$$

and a steady-state version of (39) is

$$\sigma n_i = \lambda_w(\theta_i)s_i. \tag{56}$$

Equations (55) and (56) are two equations in s and n given w, which can be thought of as implying a household's supply of searchers and worker $s(w)$ and $n(w)$. Later, when we connect the steady states of the consumer side and the firm side, we will use $\theta = a/s$ and $\sigma n = \lambda_f(\theta)a = \lambda_w(\theta)s = m$, since $\lambda_f = m/a$ and $\lambda_w = m/s$.

B Firm Problem with Standard Capital

We start with standard capital in order to explain how we model long-term contracting between firms and consumers in a simple benchmark. Section C extends this logic to the model with putty-clay capital. With standard capital, the production technology is

$$F(k_{jt}, \bar{n}_{\ell jt}, \bar{n}_{hjt}) = \left[\psi(k_{jt})^{\frac{\rho-1}{\rho}} + (1-\psi)G(\cdot)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \text{with } G(\cdot) = \left[\lambda\left(\bar{n}_{\ell jt}\right)^{\frac{\alpha-1}{\alpha}} + (1-\lambda)(\bar{n}_{hjt})^{\frac{\alpha-1}{\alpha}}\right]^{\frac{\alpha}{\alpha-1}}, \tag{57}$$

where $\bar{n}_{\ell jt}$ and $\bar{n}_{hjt}$ are given by

$$\bar{n}_{\ell jt} = \left[\sum_{i\in I_\ell} z_i(\mu_i n_{ijt})^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \text{ and } \bar{n}_{hjt} = \left[\sum_{i\in I_h} z_i(\mu_i n_{ijt})^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}. \tag{58}$$

As in the main text, it is convenient to substitute out for the aggregates $\bar{n}_{\ell jt}$ and $\bar{n}_{hjt}$ in terms of the individual inputs $\{\mu_i n_{ijt}\}$ for $i = 1, \dots, 2M$. Recall that n_{ijt} is the amount of labor supplied by a family of type i and $N_{ijt} \equiv \mu_i n_{ijt}$ is the total amount of labor of type i demanded by firm j at t. Likewise, $\mu_i a_{ijt}$ is the total amount of vacancies that firm j posts for consumers of type i at

t. We express the production function as $\tilde{F}(k, \{\mu_i n_i\})$, which in shorthand notation is

$$\tilde{F}(k, \mu_1 n_1, \ldots, \mu_M n_M; \mu_{M+1} n_{M+1}, \ldots, \mu_{2M} n_{2M}) \text{ and } \tilde{F}_{ni} = \frac{\partial \tilde{F}}{\partial(\mu_i n_1)}$$

to make clear the labor demand by the firm is the measure of workers of type j, namely, $N_{ij} = \mu_i n_{ij}$.

B.1 Initial Equilibrium without Minimum Wage

We begin with the case without the minimum wage, which corresponds to the initial steady-state we used to parameterize the model in the main text.

Profit Maximization Problem. Given an initial capital stock k_0, initial workers in the firm $\{\mu_i n_{ij0}\}$, and initial wages $\{W_{i0}\}$, each firm chooses sequences of: *i*) market tightnesses $\{\theta_{ijt}\}$ for markets for consumers of any type i, *ii*) measures of vacancies $\{\mu_i a_{ijt}\}$ to post aimed at consumers of type i, *iii*) measures of consumers of each type i to employ $\{\mu_i n_{ijt+1}\}$, *iv*) the present values of wages $\{W_{ijt+1}\}$ for these consumers, and *v*) new capital $\{k_{t+1}\}$ in order to maximize

$$\begin{gathered}\tilde{F}\left(k_{j0}, \{\mu_i n_{ij0}\}\right) - [k_{j1} - (1-\delta)k_{j0}] - \textstyle\sum_i \left[W_{i0} n_{ij0}\right] \mu_i \\ + \textstyle\sum_{t=1}^{\infty} Q_{0,t} \sum_i \left\{\tilde{F}\left(k_{jt}, \{\mu_i n_{ijt}\}\right) - [k_{jt+1} - (1-\delta)k_{jt}] - \sum_i [W_{ijt}\lambda_f(\theta_{ijt-1})a_{ijt-1} + \kappa_i a_{ijt}]\mu_i\right\},\end{gathered}$$

subject to the transition laws for total employment of each type i

$$Q_{0,t}\alpha_{ijt} : \mu_i n_{ijt+1} \le (1-\sigma)\mu_i n_{ijt} + \lambda_f(\theta_{ijt})\mu_i a_{ijt} \tag{59}$$

and a rewritten version of the participation constraints

$$Q_{0,t}\mu_i\gamma_{ijt+1} : W_{ijt+1} + V_{ijt+1} \ge \frac{\mathcal{W}_t}{\lambda_w(\theta_{ijt})} \tag{60}$$

for all j and t, where for convenience we have divided both sides of the participation constraint $\lambda_w(\theta_{ijt})(W_{ijt+1} + V_{ijt+1}) = \mathcal{W}_t$ by $\lambda_w(\theta_{ijt})$. Here $Q_{0,t}\alpha_{ijt}$ and $Q_{0,t}\mu_i\gamma_{ijt+1}$ denote the normalized multipliers on these constraints. Note that firm j should be thought as choosing the measures of vacancies and workers for each family, namely $\overline{A}_{ij} = \mu_i a_{ij}$ and $\overline{N}_{ij} = \mu_i n_{ij}$. The participation constraint is a family level constraint in which the family is choosing n_{ij}. We find it convenient not to introduce more notation and simply let the firm choose $\mu_i a_{ij}$ and $\mu_i n_{ij}$ for each i.

Gathering Terms in the Participation Constraints as in Marcet and Marimon (2019). The profit maximization problem as stated is extremely cumbersome because firms' hiring decisions impact participation constraints in all future periods. We address this issue by writing the problem in a quasi-Lagrangian form in which we gather together all the terms that enter the Lagrangian that come from the participation constraint and write them in a way the makes taking derivatives with respect to n_{ijt} convenient. To do so, we expand out the terms in V_{ijt+1} from its recursive form

$$V_{ijt+1} = \frac{u_{nit+1}}{u_{cit+1}}\left(\frac{n_{ijt+1}}{n_{it+1}}\right)^{\frac{1}{\omega}} + Q_{t+1,t+2}(1-\sigma)V_{ijt+2} \tag{61}$$

to obtain

$$V_{ijt+1} = \frac{u_{nit+1}}{u_{cit+1}}\left(\frac{n_{ijt+1}}{n_{it+1}}\right)^{\frac{1}{\omega}} + (1-\sigma)Q_{t+1,t+2}\frac{u_{nit+2}}{u_{cit+2}}\left(\frac{n_{ijt+2}}{n_{it+2}}\right)^{\frac{1}{\omega}} + (1-\sigma)^2 Q_{t+2,t+3}\frac{u_{nit+3}}{u_{cit+3}}\left(\frac{n_{ijt+3}}{n_{it+3}}\right)^{\frac{1}{\omega}} + \ldots.$$

In terms of date 0 units this rewritten participation constraint can be expanded out to

$$\mu_i\gamma_{ijt+1}: Q_{0,t+1}\frac{u_{nit+1}}{u_{cit+1}}\left(\frac{n_{ijt+1}}{n_{it+1}}\right)^{\frac{1}{\omega}} + (1-\sigma)Q_{0,t+2}\frac{u_{nit+2}}{u_{cit+2}}\left(\frac{n_{ijt+2}}{n_{it+2}}\right)^{\frac{1}{\omega}} + (1-\sigma)^2 Q_{0,t+3}\frac{u_{nit+3}}{u_{cit+3}}\left(\frac{n_{ijt+3}}{n_{it+3}}\right)^{\frac{1}{\omega}} + \ldots + Q_{0,t+1}W_{ijt+1} \geq Q_{0,t+1}\frac{\mathcal{W}_{ijt}}{\lambda_w(\theta_{ijt})}.$$

To see how we will collect terms, consider the contribution of the constraints in the first three periods $t = 0, 1, 2, \ldots,$

$$\mu_i\gamma_{ij1}: Q_{0,1}\frac{u_{ni1}}{u_{c1}}\left(\frac{n_{ij1}}{n_{i1}}\right)^{\frac{1}{\omega}} + (1-\sigma)Q_{0,2}\frac{u_{ni2}}{u_{ci2}}\left(\frac{n_{ij2}}{n_{i2}}\right)^{\frac{1}{\omega}} + (1-\sigma)^2 Q_{0,3}\frac{u_{ni3}}{u_{ci3}}\left(\frac{n_{ij3}}{n_{i3}}\right)^{\frac{1}{\omega}} + \ldots + Q_{0,1}W_{i1} - Q_{0,1}\frac{\mathcal{W}_{i0}}{\lambda_w(\theta_{ij0})} \geq 0$$

$$\mu_i\gamma_{ij2}: Q_{0,2}\frac{u_{ni2}}{u_{ci2}}\left(\frac{n_{ij2}}{n_{i2}}\right)^{\frac{1}{\omega}} + (1-\sigma)Q_{0,3}\frac{u_{ni3}}{u_{ci3}}\left(\frac{n_{ij3}}{n_{i3}}\right)^{\frac{1}{\omega}} + (1-\sigma)^2 Q_{0,4}\frac{u_{ni4}}{u_{ci4}}\left(\frac{n_{ij4}}{n_{i4}}\right)^{\frac{1}{\omega}} + \ldots + Q_{0,2}W_{i2} - Q_{0,2}\frac{\mathcal{W}_{i1}}{\lambda_w(\theta_{ij1})} \geq 0$$

$$\mu_i\gamma_{ij3}: Q_{0,3}\frac{u_{ni3}}{u_{ci3}}\left(\frac{n_{ij3}}{n_{i3}}\right)^{\frac{1}{\omega}} + (1-\sigma)Q_{0,4}\frac{u_{ni4}}{u_{ci4}}\left(\frac{n_{ij4}}{n_{i4}}\right)^{\frac{1}{\omega}} + (1-\sigma)^2 Q_{0,5}\frac{u_{ni5}}{u_{ci5}}\left(\frac{n_{ij5}}{n_{i5}}\right)^{\frac{1}{\omega}} + \ldots + Q_{0,3}W_{i3} - Q_{0,3}\frac{\mathcal{W}_{i3}}{\lambda_w(\theta_{ij2})} \geq 0.$$

The Lagrangian of the firm's profit maximization problem includes the sum of each of these terms multiplied by their associated multiplier. It is convenient to group terms by the multipliers attached to terms of the form $Q_{0,t}(u_{nit}/u_{cit})(n_{ijt}/n_{it})^{\frac{1}{\omega}}$, which conveniently groups the n_{ijt} terms for when we take derivatives of n_{ijt}. Doing so yields a sum of the form

$$\mu_i\gamma_{ij1}Q_{0,1}\left[\frac{u_{ni1}}{u_{c1}}\left(\frac{n_{ij1}}{ni_1}\right)^{\frac{1}{\omega}}\right] + \left[\mu_i\gamma_{ij2} + (1-\sigma)\mu_i\gamma_{ij1}\right]\left[Q_{0,2}\frac{u_{ni2}}{u_{c2}}\left(\frac{n_{ij2}}{ni_2}\right)^{\frac{1}{\omega}}\right]$$
$$+ \left[\mu_i\gamma_{ij3} + (1-\sigma)\mu_i\gamma_{ij2} + (1-\sigma)^2\mu_i\gamma_{ij1}\right] Q_{0,3}\left(\frac{u_{ni3}}{u_{c3}}\left(\frac{n_{ij3}}{ni_3}\right)^{\frac{1}{\omega}}\right)$$
$$+ \mu_i\gamma_{ij1}Q_{0,1}\left[W_1 - \frac{\mathcal{W}_0}{\lambda_w(\theta_{ij0})}\right] + \mu_i\gamma_{ij2}Q_{0,2}\left[W_2 - \frac{\mathcal{W}_1}{\lambda_w(\theta_{ij1})}\right] + \mu_i\gamma_{ij3}Q_{0,3}\left[W_3 - \frac{\mathcal{W}_2}{\lambda_w(\theta_{ij2})}\right] + \ldots.$$

This sum can be written more compactly as

$$M_{ij1}Q_{0,1}\frac{u_{ni1}}{u_{ci1}}\left(\frac{n_{ij1}}{n_{i1}}\right)^{\frac{1}{\omega}} + M_{ij2}Q_{0,2}\frac{u_{ni2}}{u_{ci2}}\left(\frac{n_{ij2}}{n_{i2}}\right)^{\frac{1}{\omega}} + M_{ij3}Q_{0,3}\frac{u_{ni3}}{u_{ci3}}\left(\frac{n_{ij3}}{n_{i3}}\right)^{\frac{1}{\omega}}$$
$$+ \mu_i\gamma_{ij1}Q_{0,1}\left[W_{i1} - \frac{\mathcal{W}_{i0}}{\lambda_w(\theta_{ij0})}\right] + \mu_i\gamma_{ij2}Q_{0,2}\left[W_{i2} - \frac{\mathcal{W}_{i1}}{\lambda_w(\theta_{ij1})}\right] + \mu_i\gamma_{ij3}Q_{0,3}\left[W_{i3} - \frac{\mathcal{W}_{i2}}{\lambda_w(\theta_{ij2})}\right] + \ldots,$$

where

$$M_{ij1} = \mu_i\gamma_{ij1},\ M_{ij2} = \mu_i\gamma_{ij2} + (1-\sigma)\mu_i\gamma_{ij1} = \mu_i\gamma_{ij2} + (1-\sigma)M_{ij1}.$$

More generally, we can write the law of motion for M_{ijt+1} recursively as

$$M_{ijt+1} = \mu_i\gamma_{ijt+1} + (1-\sigma)M_{ijt}, \tag{62}$$

where we define the artificial variable $M_{ij0} \equiv 0$. With this formulation, all the terms in the participation constraint for all i, j, and t in the Lagrangian are simply

$$\textstyle\sum_{t=1}^{\infty} Q_{0,t}\sum_i\left[M_{ijt}\frac{u_{nit}}{u_{cit}}\left(\frac{n_{ijt}}{n_{it}}\right)^{\frac{1}{\omega}} + \mu_i\gamma_{ijt}\left(W_{ijt} - \frac{\mathcal{W}_{ijt-1}}{\lambda_{wijt-1}}\right)\right]. \tag{63}$$

In sum, we have gathered the terms in $W_{ijt} + V_{ijt}$ from *all* the participation constraints

$$Q_{0,t}\gamma_{ijt} : W_{ijt} + V_{ijt} \geq \mathcal{W}_{it}/\lambda_w(\theta_{ijt-1})$$

and grouped together all the multipliers that affect n_{ijt} into one term.

Solving the Firm Problem. We now use the results above to solve the firm's profit maximization problem. The initial conditions include the $\mu_i n_{ij0}$ workers paid w_{i0} per period or

$$W_{i0} = w_{i0}\left[1 + (1-\sigma)Q_{0,1} + (1-\sigma)^2 Q_{0,2} + \ldots\right],$$

together with k_0 units of capital inherited from the past used in production in period 0. Here we begin by assuming it is never optimal to fire workers who are already employed. The firm chooses $\{a_{ijt}, \theta_{ijt}, W_{ijt+1}, n_{ijt+1}, k_{jt+1}\}$ for $t \geq 0$, where $\{W_{i0}, n_{ij0}, k_{j0}\}$ are the initial conditions, in order to maximize the quasi-Lagrangian

$$\begin{aligned}
&\tilde{F}\left(k_0, \{\mu_i n_{ij0}\}\right) - [k_{j1} - (1-\delta)k_{j0}] - \textstyle\sum_i \left[W_{i0} n_{ij0} + \kappa_i a_{ij0}\right]\mu_i + \sum_{t=1}^{\infty} Q_{0,t} \\
&\cdot \textstyle\sum_i \left\{\tilde{F}\left(k_{jt}, \{\mu_i n_{ijt}\}\right) - [k_{jt+1} - (1-\delta)k_{jt}] + \sum_i \left[M_{ijt}\frac{u_{nit}}{u_{cit}}\left(\frac{n_{ijt}}{n_{it}}\right)^{\frac{1}{\omega}} + \mu_i\gamma_{ijt}\left(W_{ijt} - \frac{\mathcal{W}_{ijt-1}}{\lambda_w(\theta_{ijt-1})}\right)\right]\right\} \\
&- \textstyle\sum_{t=1}^{\infty} Q_{0,t}\sum_i [W_{ijt}\lambda_f(\theta_{ijt-1}) + \kappa_i]\mu_i a_{ijt-1},
\end{aligned}$$

subject to the transition laws for total employment of each type i for all j and t,

$$Q_{0,t+1}\alpha_{ijt+1} : \mu_i n_{ijt+1} \leq (1-\sigma)\mu_i n_{ijt} + \lambda_f(\theta_{ijt})\mu_i a_{ijt}.$$

As a side condition (rather than a constraint), we have the law of motion for the sum of the multipliers

$$M_{ijt+1} = \mu_i\gamma_{ijt+1} + (1-\sigma)M_{ijt} \tag{64}$$

with $M_{j0} = 0$ and $\theta_{jt} = a_{jt}/s_{jt}$. We now derive the FOCs of this problem.

The FOC for k_{jt+1} is standard. Using $Q_{t,t+1} = Q_{0,t+1}/Q_{0,t}$, we have

$$Q_{t,t+1}[\tilde{F}_{kjt+1} + (1-\delta)] = 1. \tag{65}$$

The FOC for the present value of wages W_{ijt+1} is

$$W_{ijt+1} : \lambda_f(\theta_{ijt})a_{ijt} = \gamma_{ijt+1} \text{ for } t = 0, 1, \ldots. \tag{66}$$

The FOC for next period's employment n_{ijt+1}, after dividing by μ_i, is

$$n_{ijt+1} : Q_{0,t+1}\alpha_{ijt+1} = Q_{0,t+1}\tilde{F}_{nijt+1} + Q_{0,t+1}\frac{M_{ijt+1}}{\mu_i}\frac{1}{u_{cit+1}}u_{nit+1}\frac{\partial\left[(n_{ijt+1}/n_{it+1})^{\frac{1}{\omega}}\right]}{\partial n_{ijt+1}} + (1-\sigma)Q_{0,t+2}\alpha_{ijt+2},$$

where we have used that from the household's preferences (44) $\partial n_{it+1}/\partial n_{ijt+1} = (n_{ijt+1}/n_{it+1})^{\frac{1}{\omega}}$. Further dividing by $Q_{0,t+1}$, we obtain

$$n_{ijt+1} : \alpha_{ijt+1} = \tilde{F}_{nijt+1} + \frac{M_{ijt+1}}{\mu_i}\frac{u_{nit+1}}{u_{cit+1}}\frac{\partial\left[(n_{ijt+1}/n_{it+1})^{\frac{1}{\omega}}\right]}{\partial n_{ijt+1}} + (1-\sigma)Q_{t+1,t+2}\alpha_{ijt+2}. \tag{67}$$

Note that under our assumption that the number of firms J is sufficiently large, we have

$$\frac{\partial\left[(n_{ijt}/n_{it})^{\frac{1}{\omega}}\right]}{\partial n_{ijt}} = \frac{1}{n_{it}^{1/\omega}}\frac{\partial\left(n_{ijt}^{\frac{1}{\omega}}\right)}{\partial n_{ijt}} = \frac{1}{n_{it}^{1/\omega}}\frac{1}{\omega}n_{ijt}^{\frac{1}{\omega}-1} = \frac{1}{\omega n_{ijt}}\left(\frac{n_{ijt}}{n_{it}}\right)^{\frac{1}{\omega}}. \tag{68}$$

Substituting (68) into (67) and evaluating it in period t for n_{ijt} gives the recursive form for α_{ijt}

$$n_{ijt} : \alpha_{ijt} = \tilde{F}_{nijt} + \frac{M_{ijt}}{\mu_i}\frac{u_{nit}}{u_{cit}}\frac{1}{\omega n_{ijt}}\left(\frac{n_{ijt}}{n_{it}}\right)^{\frac{1}{\omega}} + (1-\sigma)Q_{t,t+1}\alpha_{ijt+1}. \tag{69}$$

To help interpret this equation, recurse it forward to obtain

$$n_{ijt} : \alpha_{ijt} = \textstyle\sum_{k=0}^{\infty}(1-\sigma)^k Q_{t,t+k}\left[\tilde{F}_{nijt+k} + \frac{M_{jt+k}}{\mu_i}\frac{u_{nit+k}}{u_{cit+k}}\frac{1}{\omega n_{ijt+k}}\left(\frac{n_{ijt+k}}{n_{it+k}}\right)^{\frac{1}{\omega}}\right]. \tag{70}$$

Thus, the value of an incremental measure of workers of type i for firm j at t is the present value of all the output that these workers produce minus the increase in disutility that such workers generate for their family. Here through the FOC for n_{ijt+1}, firms understand that by hiring more workers of type i, they are increasing the disutility of such a family through the term $\partial\left[(n_{ijt+1}/n_{it+1})^{\frac{1}{\omega}}\right]/\partial n_{ijt+1}$ and through the sum of past multipliers in M_{ijt+1}, which represents the disutility of all the previously-hired workers who are still employed by the firm.

The FOC for vacancies a_{ijt} is

$$a_{ijt} : Q_{0,t+1}\lambda_f(\theta_{ijt})W_{ijt+1} + Q_{0,t}\kappa = Q_{0,t+1}\lambda_f(\theta_{ijt})\alpha_{ijt+1} \text{ for } t = 0, 1, \ldots$$

or

$$a_{ijt} : \frac{\kappa_i}{Q_{t,t+1}\lambda_f(\theta_{ijt})} = \alpha_{ijt+1} - W_{ijt+1}, \tag{71}$$

which will be the break-even or *free-entry condition for vacancy posting*. This condition implies that the cost of posting vacancies is equal to the discounted sum of the output that newly hired workers will produce minus the increase in disutility that these workers will generate for a family of type i, minus the wage bill from hiring them.

We now derive, and then simplify, the FOC for vacancies θ_{ijt}:

$$W_{ijt+1}a_{ijt}\lambda_f'(\theta_{ijt}) = \gamma_{ijt+1}\frac{\lambda_w'(\theta_{ijt})}{\lambda_w(\theta_{ijt})^2}\mathcal{W}_{it} + \alpha_{jt+1}a_{ijt}\lambda_f'(\theta_{ijt}) \tag{72}$$

$$W_{ijt+1}a_{ijt}\lambda_f'(\theta_{ijt}) = \lambda_f(\theta_{ijt})a_{ijt}\frac{\lambda_w'(\theta_{ijt})}{\lambda_w(\theta_{ijt})^2}\lambda_w(\theta_{ijt})\left(W_{ijt+1} + V_{ijt+1}\right) + \alpha_{ijt+1}a_{ijt}\lambda_f'(\theta_{ijt}) \tag{73}$$

$$W_{ijt+1} = \frac{\lambda_f(\theta_{ijt})}{\lambda_f'(\theta_{ijt})}\frac{\lambda_w'(\theta_{ijt})}{\lambda_w(\theta_{ijt})}\left(W_{ijt+1} + V_{ijt+1}\right) + \alpha_{ijt+1} \tag{74}$$

$$W_{ijt+1} = -\frac{1-\eta}{\eta}\left(W_{ijt+1} + V_{ijt+1}\right) + \alpha_{ijt+1} = -\frac{1}{\eta}W_{ijt+1} + W_{ijt+1} - \frac{1-\eta}{\eta}V_{ijt+1} + \alpha_{ijt+1} \tag{75}$$

$$\tag{76}$$

and so

$$\frac{W_{ijt+1}}{\eta} = -\frac{1-\eta}{\eta}V_{ijt+1} + \alpha_{ijt+1} \text{ so } W_{ijt+1} = -(1-\eta)V_{ijt+1} + \eta\alpha_{ijt+1}. \tag{77}$$

In (73), we substituted for $\gamma_{ijt+1} = \lambda_f(\theta_{ijt})a_{ijt}$ and $\mathcal{W}_{it} = \lambda_w(\theta_{ijt})\left(W_{ijt+1} + V_{ijt+1}\right)$, and in (75) we used that

$$\frac{\lambda_f}{\lambda'_f}\frac{\lambda'_w}{\lambda_w} = -\frac{\lambda_f}{\eta\lambda_f/\theta}\frac{(1-\eta)\lambda_w/\theta}{\lambda_w} = -\frac{1-\eta}{\eta}. \tag{78}$$

This equation follows from the facts that $\lambda_f = B\theta^{-\eta}$ so $\lambda'_f = -B\eta\theta^{-\eta-1} = -B\eta\lambda_f/\theta < 0$ and $\lambda_w = B\theta^{1-\eta}$ so $\lambda'_w = B(1-\eta)\theta^{-\eta} = (1-\eta)\lambda_w/\theta$.

We now impose symmetry across firms j. In this case, $(n_{ijt+k}/n_{it+k})^{\frac{1}{\omega}} = 1$ and the equations (65), (61), (71), and (77) above become

$$\begin{aligned}
&Q_{t,t+1}[\tilde{F}_{kt+1} + (1-\delta)] = 1\\
&V_{it+1} = \frac{u_{nit+1}}{u_{cit+1}} + (1-\sigma)Q_{t+1,t+2}V_{it+2}\\
&\alpha_{it+1} = \tilde{F}_{nit+1} + \frac{M_{it+1}}{\mu_i}\frac{u_{nit+1}}{u_{cit+1}}\frac{1}{\omega n_{it+1}} + (1-\sigma)Q_{t+1,t+2}\alpha_{it+2},
\end{aligned}$$

with

$$W_{it+1} = w_{it+1}\left[1 + (1-\sigma)Q_{t+1,t+2} + (1-\sigma)^2 Q_{t+1,t+3} + \ldots\right]. \tag{79}$$

Later on, it will be convenient to use the equations

$$a_{it} : W_{it+1} + \frac{\kappa}{Q_{t,t+1}\lambda_f(\theta_{it})} = \alpha_{it+1} \tag{80}$$

$$\theta_{it} : W_{it+1} = -(1-\eta)V_{it+1} + \eta\alpha_{it+1}, \tag{81}$$

and substitute W_{it+1} from (81) into the FOC for vacancies a_{it} in (80) to obtain

$$\frac{\kappa}{Q_{t,t+1}\lambda_f(\theta_{it})} = (1-\eta)(\alpha_{it+1} + V_{it+1}). \tag{82}$$

Alternatively, it will sometimes be useful to write the equation as

$$\frac{\kappa}{Q_{t,t+1}\lambda_f(\theta_{it})} = \alpha_{it+1} - W_{it+1}. \tag{83}$$

Both forms of the free-entry condition for vacancy posting will be useful.

Summary of Symmetric Equilibrium. To summarize, the Euler equation for capital is

$$Q_{t,t+1}[\tilde{F}_{kt+1} + (1-\delta)] = 1.$$

The optimal vacancy-posting condition is

$$a_{it} : \frac{\kappa}{Q_{t,t+1}\lambda_f(\theta_{it})} = (1-\eta)(\alpha_{it+1} + V_{it+1}). \tag{84}$$

The market tightness condition implies

$$\theta_{it} : W_{it+1} = -(1-\eta)V_{it+1} + \eta\alpha_{it+1}, \tag{85}$$

where α_{it} and V_{it} are given recursively by

$$\alpha_{it} = \left[\tilde{F}_{nit} + \frac{M_{it}}{\mu_i}\frac{u_{nit}}{u_{cit}}\frac{1}{\omega n_{it}}\right] + (1-\sigma)Q_{t,t+1}\alpha_{it+1} \tag{86}$$

and

$$V_{it} = \frac{u_{nit}}{u_{cit}} + (1-\sigma)Q_{t,t+1}V_{it+1}. \tag{87}$$

We also have the equations for the multipliers $M_{it+1} = \mu_i\gamma_{it+1} + (1-\sigma)M_{it}$ and the FOC for wages

$$W_{it+1} : \lambda_f(\theta_{it})a_{ijt} = \gamma_{it+1} \text{ for } t = 0, 1, \ldots.$$

Note the above two equations imply that M_{it+1} has the law of motion

$$M_{it+1} = \mu_i\lambda_f(\theta_{it})a_{it} + (1-\sigma)M_{it}, \tag{88}$$

which will be useful later on.

Symmetric Steady State. We now characterize these equations in steady state, which leads to even simpler conditions. In steady state, θ_i, a_i, n_i, M_i, α_i, and γ_i are all constant over time. The capital Euler equation becomes

$$\tilde{F}_k = \frac{1}{\beta} - (1-\delta) = r - \delta \text{ where } 1 + r \equiv \frac{1}{\beta}.$$

The law of motion for employment becomes

$$n_i = (1-\sigma)n_i + \lambda_f(\theta_i)a_i \text{ so } \sigma n_i = \lambda_f(\theta_i)a_i,$$

which then from (88) implies

$$\sigma M_i = \mu_i\lambda_f(\theta_i)a_i = \mu_i\sigma n_i \text{ so } M_i = \mu_i n_i \text{ so } \frac{M_i}{\mu_i} = n_i. \tag{89}$$

The marginal disutility of work becomes (61) or

$$V_i = \frac{u_{ni}/u_{ci}}{1-\beta(1-\sigma)}, \tag{90}$$

which then gives (86) or

$$\alpha_i = \tilde{F}_{ni} + \frac{M_i}{\mu_i}\frac{u_{ni}}{u_{ci}}\frac{1}{\omega n_i} + \beta(1-\sigma)\alpha_i.$$

Using (89), this equation can be manipulated to obtain

$$\alpha_i = \frac{\tilde{F}_{ni} + n_i\frac{u_{ni}}{u_{ci}}\frac{1}{\omega n_i}}{1-\beta(1-\sigma)} = \frac{\tilde{F}_{ni} + \frac{1}{\omega}\frac{u_{ni}}{u_{ci}}}{1-\beta(1-\sigma)}. \tag{91}$$

Hence, we obtain a version of the free-entry condition for vacancy posting with wages substituted out, namely,

$$\frac{\kappa}{\beta\lambda_f(\theta_i)} = (1-\eta)(\alpha_i + V_i),$$

which after substituting from (90) and (91) gives

$$\frac{\kappa}{\beta\lambda_f(\theta_i)} = (1-\eta)\frac{\tilde{F}_{ni} + \frac{u_{ni}}{u_{ci}}(1+\frac{1}{\omega})}{1-\beta(1-\sigma)}.$$

There is also an alternative version of this condition that does not substitute out for wages, which is the steady state version of (80),

$$\frac{\kappa}{\beta\lambda_f(\theta_i)} = \frac{\tilde{F}_{ni} + \frac{1}{\omega}\frac{u_{ni}}{u_{ci}} - w_i}{1-\beta(1-\sigma)},$$

where we used (91) and $W_i = w_i/[1-\beta(1-\sigma)]$.

We now derive an expression for flow wages w_i in a symmetric steady state. Start with the optimal tightness condition $W_i = -(1-\eta)V_i + \eta\alpha_i$ and use the results

$$W_i = \frac{w_i}{1-\beta(1-\sigma)},\ V_i = \frac{u_{ni}/u_{ci}}{1-\beta(1-\sigma)} \text{ and } \alpha_i = \frac{\tilde{F}_{ni} + \frac{1}{\omega}\frac{u_{ni}}{u_{ci}}}{1-\beta(1-\sigma)}$$

to obtain

$$w_i = \eta\left(\tilde{F}_{ni} + \frac{1}{\omega}\frac{u_{ni}}{u_{ci}}\right) - (1-\eta)\frac{u_{ni}}{u_{ci}}.$$

To further simplify these expressions, we consider preferences in the spirit of Greenwood, Hercowitz and Huffman (1988), which we use in our quantitative exercises, $U(c_i - h(s_i) - v(n_i))$. Note that the free-entry condition for vacancy posting with wages substituted out is

$$\frac{\kappa}{\beta\lambda_f(\theta_i)} = (1-\eta)(\alpha_i + V_i)$$

or substituting for α_i and V_i and using $1+r = 1/\beta$,

$$\frac{\kappa}{\lambda_f(\theta_i)} = \frac{(1-\eta)}{r+\sigma}\left[\tilde{F}_{ni} - (1+\frac{1}{\omega})v'(n)\right].$$

The free-entry condition for vacancies with wages left in is

$$\frac{\kappa}{\lambda_f(\theta_i)} = \frac{\tilde{F}_{ni} - \frac{v'(n)}{\omega} - w_i}{r+\sigma}, \tag{92}$$

where wages are given by

$$w_i = \eta\left[\tilde{F}_{ni} - \frac{v'(n)}{\omega}\right] + (1-\eta)v'(w).$$

B.2 Firm Problem with Minimum Wage

We now consider the firm's problem once the minimum wage is unexpectedly introduced at time $t = 0$, starting from the initial steady state characterized above. We assume that all firing or quitting of the existing workers takes place in period 0. In particular, wage offers after the minimum wage is introduced, namely for $t \geq 1$, must satisfy

$$W_{it} \equiv d_t w_{it} \geq d_t \underline{w} = \underline{W}_t \text{ where } d_t = (1 + Q_{t,t+1} + \ldots).$$

For existing workers, the wage is set to be the larger of the min wage $\underline{w}$ and the initial steady state wage w_i^s, namely, $\overline{w}_i = \max\{\underline{w}, w_i^s\}$. The workers employed in the initial steady state who are retained in period 0 work in that period and are paid

$$[1 + Q_{0,1} + \ldots]\,\overline{w}_i \equiv d_0\overline{w}_i \text{ where } d_t = (1 + Q_{t,t+1} + \ldots) \text{ for } t = 0, 1, \ldots.$$

Profit Maximization Problem. The household problem is unchanged, so we focus on the firm problem. In period 0, there are $\{n^s_{ij}\}$ workers employed. Firm j chooses to retain n^r_{ij0} of them and pay them $\overline{w}_i = \max\{\underline{w}, w^s_i\}$ from then on with discounted value $\overline{W}_{i0} = d_0\overline{w}_i$ subject to the constraint that retained workers are a subset of the workers employed in the initial steady state, that is, $n^r_{ij0} \leq n^s_{ij}$. The participation constraint for workers hired in period 0 who start working in period 1 is

$$Q_{0,1}\gamma_{ij1}: \quad W_{ij1} + V_{ij1} \geq \frac{\mathcal{W}_{i0}}{\lambda_w(\theta_{ij0})}.$$

In period 0, the firm also chooses k_{j1},a_{ij0}, θ_{ij0}, and W_{ij1}, where all new hires from period 1 on must be paid at least the minimum wage, that is,

$$W_{ijt} \geq \underline{W}_t = d_t\underline{w} \quad \text{for } t \geq 1.$$

For period $t \geq 1$, it chooses $\{\theta_{ijt}, W_{ijt+1}, n_{ijt+1}, a_{ijt}, k_{jt+1}\}$. The profit maximization problem is

$$\tilde{F}\left(k_{j0}, \{\mu_i n^r_{ij0}\}\right) - [k_{j1} - (1-\delta)k_{j0}] - \textstyle\sum_i \overline{W}_{i0} n^r_{ij0}\mu_i$$
$$+ \textstyle\sum_{t=1}^{\infty} Q_{0,t} \sum_i \left\{\tilde{F}\left(k_{jt}, \{\mu_i n_{ijt}\}\right) - [k_{jt+1} - (1-\delta)k_{jt}]\right\}$$
$$+ \textstyle\sum_{t=1}^{\infty} Q_{0,t} \sum_i \left\{M_{ijt} \frac{u_{nit}}{u_{cit}} \left(\frac{n_{ijt}}{n_{it}}\right)^{\frac{1}{\omega}} + \mu_i\gamma_{ijt}\left[W_{ijt} - \frac{\mathcal{W}_{ijt-1}}{\lambda_w(\theta_{ijt-1})}\right] - \sum_i [W_{ijt}\lambda_f(\theta_{ijt-1}) + \kappa_i]\mu_i a_{ijt-1}\right\},$$

subject to the constraints

$$\begin{aligned}
&\alpha^r_{ij0} : \mu_i n^r_{ij0} \leq \mu_i n^s_{ij},\\
&Q_{0,1}\alpha_{ij1} : \mu_i n_{ij1} \leq (1-\sigma)\mu_i n^r_{ij0} + \lambda_f(\theta_{ij0})\mu_i a_{ij0}\\
&Q_{0,t+1}\alpha_{ijt+1} : \mu_i n_{ijt+1} \leq (1-\sigma)\mu_i n_{ijt} + \lambda_f(\theta_{ijt})\mu_i a_{ijt} \text{ for } t \geq 1\\
&\mu_i\eta_{ij0} : W_{ij0} \geq \overline{W}_{i0} = d_0\overline{w}_i\\
&Q_{0,t+1}\mu_i\eta_{ijt+1} : W_{ijt+1} \geq \underline{W}_{t+1} = d_{t+1}\underline{w}\\
&Q_{0,t}\psi_{ijt} : \mu_i a_{ijt} \geq 0,
\end{aligned}$$

where $M_{ijt+1} = \gamma_{ijt+1} + (1-\sigma)M_{ijt}$ with $M_{ij0} = 0$ and we have defined $\overline{W}_{i0} \equiv d_0\overline{w}_i$.

Retention and Firing Decisions in Initial Period. The retention and firing decisions in period 0 embedded in n^r_{ij0} are reflected in the multipliers $\alpha^r_{ij0} \geq 0$, given by

$$\alpha^r_{ij0} = (\tilde{F}_{nij0} - \overline{W}_{i0}) + Q_{0,1}(1-\sigma)\alpha_{ij1}.$$

If at a candidate allocation in which all initial workers of this type are retained, the resulting $\alpha^r_{ij0} < 0$, then this candidate allocation is not the solution. Instead, some workers of that type are fired and the number of workers who are retained satisfy

$$0 = (\tilde{F}_{nij0} - \overline{W}_{i0}) + Q_{0,1}(1-\sigma)\alpha_{ij1}.$$

All Other Decisions. We now characterize the optimal choices of $\{\theta_{ijt}, W_{ijt+1}, n_{ijt+1}, a_{ijt}, k_{jt+1}\}$ for $t = 0$ onwards (the conditions are the same for $t = 0$ and $t \geq 1$). The FOC for capital is still

$$k_{jt+1} : 1 = Q_{t,t+1}\left[\tilde{F}_{kjt+1} + (1-\delta)\right]. \tag{93}$$

The FOC for labor at $t \geq 1$ is

$$n_{ijt+1} : Q_{0,t+1}\alpha_{ijt+1} = Q_{0,t+1}\left[\tilde{F}_{nijt+1} + M_{ijt+1}\frac{u_{nit+1}}{u_{cit+1}}\underbrace{\frac{1}{\omega n_{ijt+1}}\left(\frac{n_{ijt+1}}{n_{it+1}}\right)^{\frac{1}{\omega}}}_{\partial[(n_{ijt+1}/n_{it+1})^{\frac{1}{\omega}}]}\right] + (1-\sigma)Q_{0,t+2}\alpha_{ijt+2}. \quad (94)$$

We can write this equation more simply as

$$n_{ijt} : \alpha_{ijt} = \left[\tilde{F}_{nijt} + M_{ijt}\frac{u_{nit}}{u_{cit}}\frac{1}{\omega n_{ijt}}\left(\frac{n_{ijt}}{n_{it}}\right)^{\frac{1}{\omega}}\right] + (1-\sigma)Q_{t,t+1}\alpha_{ijt+1}, \quad (95)$$

or, letting $A_{ijt} \equiv \tilde{F}_{nijt} + M_{ijt}\frac{u_{nit}}{u_{cit}}\frac{1}{\omega n_{ijt}}\left(\frac{n_{ijt}}{n_{it}}\right)^{\frac{1}{\omega}}$, as

$$\alpha_{ijt} = A_{ijt} + (1-\sigma)Q_{t,t+1}\alpha_{ijt+1} = \textstyle\sum_{k=0}^{\infty}(1-\sigma)^k Q_{t,t+k}A_{ijt+k},$$

after recursing it out. Here, α_{ijt} is the present value of production of a worker net of the marginal disutility of all other workers of families of type i in the current period as well as in any previous period, captured by the term $M_{ijt} = (1-\sigma)M_{ijt-1} + \gamma_{ijt}$. Recall the firm is actually hiring $N_{ijt+1} = \mu_i n_{ijt+1}$ workers, so it is hiring a marginal worker from every family of type i. Thus,

$$\tilde{F}_{nijt} \equiv \frac{\partial F}{\partial \bar{n}_{it}}\frac{\partial \bar{n}_{it}}{\partial N_{ijt}}\frac{\partial N_{ijt}}{\partial n_{ijt}} \text{ and } \frac{\partial N_{ijt}}{\partial n_{ijt}} = \mu_i.$$

For later, it will be convenient to write

$$\alpha_{ij1} = \textstyle\sum_{k=0}^{\infty}(1-\sigma)^k Q_{1,k+1}A_{ijk+1} \text{ where } A_{ijt} \equiv \tilde{F}_{nijt} + M_{ijt}\frac{u_{nit}}{u_{cit}}\frac{1}{\omega n_{ijt}}\left(\frac{n_{ijt}}{n_{it}}\right)^{\frac{1}{\omega}}.$$

Now combining (95) with $\alpha^r_{ij0} = (\tilde{F}_{nij0} - \overline{W}_{i0}) + Q_{0,1}(1-\sigma)\alpha_{ij1}$ gives

$$\alpha^r_{ij0} = (\tilde{F}_{nij0} - \overline{W}_{i0}) + (1-\sigma)Q_{0,1}A_{ij1} + (1-\sigma)^2 Q_{0,2}A_{ij2} + \dots.$$

Finally, if $\alpha^r_{ij0} > 0$, then

$$\tilde{F}_{nij0} + (1-\sigma)Q_{0,1}\left[\tilde{F}_{nij1} + M_{ij1}\frac{u_{ni1}}{u_{ci1}}\frac{1}{\omega n_{ij1}}\left(\frac{n_{ij1}}{n_{i1}}\right)^{\frac{1}{\omega}}\right] + (1-\sigma)^2 Q_{0,2}\left[\tilde{F}_{nij2} + M_{ij2}\frac{u_{ni2}}{u_{ci2}}\frac{1}{\omega n_{ij2}}\left(\frac{n_{ij2}}{n_{i2}}\right)^{\frac{1}{\omega}}\right] + \dots \geq \overline{W}_{i0}. \quad (96)$$

To interpret these conditions, consider first the competitive case with $1/\omega = 0$ and focus on a worker that is employed by the firm in the initial steady state. Then, this condition states that the present value of the output of this worker (the left side) equals the cost of paying this worker (the right side). If this worker produces more than this worker costs, then the firm finds it optimal to retain the worker. Otherwise, the firm fires the worker.

Condition (96) is the exact analog of this case but with $1/\omega > 0$, which requires accounting for the negative impact of this marginal worker on the disutility of working at this firm of all inframarginal workers, which forces the firm to increase the wages of current and future workers to offset this increased disutility. The FOC for wages is

$$W_{ijt+1} : \lambda_f(\theta_{ijt})a_{ijt} = \gamma_{ijt+1} + \eta_{ijt+1} \text{ so } \gamma_{ijt+1} = \lambda_f(\theta_{ijt})a_{ijt} - \eta_{ijt+1} \text{ or } \eta_{ijt+1} = \lambda_f(\theta_{ijt})a_{ijt} - \gamma_{ijt+1}. \quad (97)$$

This present value of wages will be paid to $\lambda_f(\theta_{ijt})a_{ijt}$ additional workers who start working in $t+1$. Paying these wages relaxes the participation constraint γ_{ijt+1} for workers who start working and getting paid in $t+1$—if the minimum wage binds at $t+1$, it relaxes this constraint as well. We maintain throughout that $a_{ijt} > 0$ for $t \geq 1$. We impose this at $t = 0$ because otherwise, the firm might would try to set $a_{ij0} < 0$, pay negative vacancy posting costs κa_{ij0} and have fewer workers at $t = 1$. For vacancies a_{ijt}, we also have the complementary slackness conditions

$$\psi_{ijt} a_{ijt} = 0, \tag{98}$$

along with the FOC for vacancies a_{ijt} for $t \geq 0$,

$$a_{ijt} : \frac{\kappa}{Q_{t,t+1}\lambda_f(\theta_{ijt})} = \alpha_{ijt+1} - W_{ijt+1} + \psi_{ijt} \text{ for } t \geq 0. \tag{99}$$

Here, the left side is the expected cost of hiring a worker. On the right side, the first term α_{ijt+1} is the present value of output of that worker minus the increased disutility of work of workers of the same type i and the term W_{ijt+1} is the present value of wages paid to that worker. Now, if

$$\frac{\kappa}{Q_{t,t+1}\lambda_f(\theta_{ijt})} > \alpha_{ijt+1} - W_{ijt+1},$$

then the cost of attracting a worker is greater than the benefit. Hence, $\psi_{ijt} > 0$ and $a_{ijt} = 0$.

The FOC for market tightness θ_{ijt} is for $t \geq 0$

$$\theta_{ijt} : \lambda_f'(\theta_{ijt}) a_{ijt} W_{ijt+1} = \gamma_{ijt+1} \frac{\mathcal{W}_{it}\lambda_w'(\theta_{ijt})}{\lambda_w(\theta_{ijt})^2} + \alpha_{ijt+1}\lambda_f'(\theta_{ijt}) a_{ijt} \text{ for } t \geq 0.$$

Substitute for $\mathcal{W}_{it} = \lambda_w(\theta_{ijt})\left(W_{ijt+1} + V_{ijt+1}\right)$ in the above to get

$$\theta_{ijt} : \lambda_f'(\theta_{ijt}) a_{ijt} W_{ijt+1} = \gamma_{ijt+1} \frac{\lambda_w'(\theta_{ijt})}{\lambda_w(\theta_{ijt})}\left(W_{ijt+1} + V_{ijt+1}\right) + \alpha_{ijt+1}\lambda_f'(\theta_{ijt}) a_{ijt} \text{ for } t \geq 0.$$

Now divide by $\lambda_f(\theta_{ijt})$ to obtain

$$\theta_{ijt} : W_{ijt+1} a_{ijt} = \gamma_{ijt+1} \frac{\lambda_w'(\theta_{ijt})}{\lambda_f'(\theta_{ijt})\lambda_w(\theta_{ijt})}\left(W_{ijt+1} + V_{ijt+1}\right) + \alpha_{ijt+1} a_{ijt} \text{ for } t \geq 0. \tag{100}$$

If $\lambda_f = \theta^{-\eta}$, then $\lambda_f' = -\eta\theta^{-\eta-1} = -\eta\lambda_f/\theta < 0$ and $\lambda_w = \theta^{1-\eta}$ so $\lambda_w' = (1-\eta)\theta^{-\eta} = (1-\eta)\lambda_w/\theta$ and

$$\frac{1}{\lambda_f'}\frac{\lambda_w'}{\lambda_w} = -\frac{\theta}{\eta\lambda_f}\frac{1-\eta}{\theta} = -\frac{1}{\lambda_f}\frac{1-\eta}{\eta},$$

which when substituted into (100) gives

$$\theta_{ijt} : W_{ijt+1} a_{ijt} = -\frac{\gamma_{ijt+1}}{\lambda_f(\theta_{ijt})}\frac{1-\eta}{\eta}\left(W_{ijt+1} + V_{ijt+1}\right) + \alpha_{ijt+1} a_{ijt} \text{ for } t \geq 0. \tag{101}$$

Next, multiply (99) by a_{ijt} and rearrange to get

$$a_{ijt} : W_{ijt+1} a_{ijt} + \frac{\kappa a_{ijt}}{Q_{t,t+1}\lambda_f(\theta_{ijt})} = \alpha_{ijt+1} a_{ijt} \; + \psi_{ijt} a_{ijt} \text{for } t \geq 0,$$

but by the complementary slackness condition (98), this becomes

$$a_{ijt} : W_{ijt+1}a_{ijt} + \frac{\kappa a_{ijt}}{Q_{t,t+1}\lambda_f(\theta_{ijt})} = \alpha_{ijt+1}a_{ijt} \text{ for } t \geq 0. \tag{102}$$

Summary of Symmetric Equilibrium. To summarize, the FOC for capital is

$$k_{t+1} : 1 = Q_{t,t+1}\big[\tilde{F}_{kt+1} + (1-\delta)\big].$$

The FOC for labor n_{i0} at $t = 0$ and those for labor n_{it} at $t > 1$ are, respectively,

$$\alpha_{i0} = (\tilde{F}_{ni0} - \overline{W}_{i0}) + Q_{0,1}(1-\sigma)\alpha_{i1}$$

and

$$\alpha_{it} = \left[\tilde{F}_{nit} + M_{it}\frac{u_{nit}}{u_{cit}}\frac{1}{\omega n_{it}}\right] + (1-\sigma)Q_{t,t+1}\alpha_{it+1}.$$

The FOC for vacancy posting for all $t \geq 0$ is

$$\frac{\kappa}{Q_{t,t+1}\lambda_f(\theta_{it})} = \alpha_{it+1} - W_{it+1} + \psi_{it},$$

together with the complementary slackness condition $\psi_{it}a_{it} = 0$. The FOC for market tightness is

$$W_{it+1}a_{it} = -\frac{\gamma_{it+1}}{\lambda_f(\theta_{it})}\frac{1-\eta}{\eta}(W_{it+1} + V_{it+1}) + \alpha_{it+1}a_{it} \text{ for } t \geq 0.$$

The FOC for wages and associated complementary slackness condition for $W_{it} \geq \overline{W}_{i0}$ are $\eta_{it+1} = \lambda_f(\theta_{it})a_{it} - \gamma_{it+1}$ and $\eta_{it+1}(W_{it+1} - \underline{W}_t) = 0$. Finally, the evolution of M_{it} is described by

$$M_{it+1} = \gamma_{it+1} + (1-\sigma)M_{it} \text{ with } M_{i0} = 0.$$

C Firm Problem with Putty-Clay Capital

Here we derive the solution to a firm's problem in the case of putty-clay capital. Since the steady state of the putty-clay model is the same as that of the model with standard capital, we focus the discussion on the transition path after the introduction of the minimum wage—the cases in which we introduce the EITC and a general tax and transfer system are analogous. We proceed in three main steps. First, in subsection C.1 we set up and give the first-order conditions for the original version of the problem in which the distribution of capital types is a state variable.

Second, in subsection C.2 we first consider a *restricted disaggregate problem* in which we simply impose that that the firm must fully utilize all capital and keeps all workers that it has hired, including the incumbent workers from the initial steady state with no minimum wage. We then show that this restricted disaggregate problem is equivalent to a *restricted aggregate problem*, which imposes these same utilization and "keeping-worker" constraints but only records aggregate output and the aggregate employment of each worker type as aggregate state variables rather than the distribution of capital types. Hence, it is much easier to solve.

Third, we use the solution of the restricted aggregate problem to construct a *candidate* solution for the original problem. We then construct the multipliers for the original problem at our candidate solution using the first-order conditions from the original problem. It immediately follows that the necessary and sufficient conditions for the solution to the restricted aggregate problem to be the solution of the original problem are simply that these multipliers have the correct sign. In our algorithm, we verify these conditions for our parameter values. In practice, we find that they do for minimum wages less than \$35 per hour.

Note that in this algorithm we need only ever compute the solution to a relatively easy problem, namely, the restricted aggregate problem with few aggregate state variables—namely, aggregate output and the aggregate employment of each worker type. This problem has neither constraints for which we have to guess the binding pattern nor constraints that occasionally bind.

C.1 The Original Firm Problem

Consider a firm that starts in an original steady state in which there is no minimum wage in the sense that the initial conditions for capital, wages, and incumbent workers come from that steady state. Given these conditions, we suppose that there is the surprise introduction of a permanent minimum wage. Here we show how we solve that problem.

The firm's initial conditions are $\{W_{i0}, n^s_{ij}, k_{j0}(v^s)\}$ and $k_{j0}(v) = k^s_{j0}$ for $v = v^s_0$ and $k_{j0}(v) = 0$ for $v \neq v^s_0$, since only one type of capital is used in the steady state. Note that since the initial labor and capital came from a steady state, these initial conditions satisfy the Leontief constraints

$$k_{j0} = n_{ij0}(v^s)/v^s_i \text{ for all } i.$$

That is, the firm has exactly the right number of workers of each type to run the one type of capital that it has at full capacity. In the firm's problem, we drop the max's from the Leontief technology and impose them as inequality constraints. We also allow firms to fire the incumbent workers in the initial steady state by adding a variable $n^r_{ij0}(v^s)$ which represents non-fired or *retained* initial workers and a constraint

$$\mu_i n^r_{ij0}(v^s) \leq \mu_i n^s_{ij}(v^s).$$

We also define the variables $k^a_{jt}(v)$ which represents the capital of type v *actively* used in production which might be less than than the total available capital $k_{jt}(v)$ of that type and the variable labor $n^a_{ijt}(v)$ which represents the *actively* used labor necessary to run the actively used capital of type v at full capacity in that

$$\frac{n^a_{ij0}(v^s)}{v^s_i} = k^a_0(v^s).$$

We also introduce the inequality constraint

$$k^a_t(v) \leq k_t(v)$$

that allows the firm to leave capital of any type v idle, the constraint

$$\mu_i n^a_{ij0}(v^s) \leq \mu_i n^r_{ij}(v^s)$$

that allows the firm to leave some of its retained workers at $t = 0$ idle, and the constraint

$$\int_v \mu_i n^a_{ijt}(v) dv \leq \mu_i n_{ijt} \text{ for } t \geq 1$$

that allows the firm to leave some of its workers idle at any $t \geq 1$.

Given the initial conditions, the firm chooses sequences $\{k^a_{j0}(v), \{n^a_{ij0}(v^s)\}, a_{ij0}\}$ for $t=0$ and

$\{k^a_{jt}(v), \{n^a_{ijt}(v)\}, a_{ijt}, k_{jt+1}(v), \theta_{ijt}, W_{ijt+1}, n_{ijt+1}\}$ for $t>0$ to solve the quasi-Lagrangian problem

$$\max \tilde{F}(k^a_{j0}(v^s), \{\mu_i n^a_{ij0}(v^s)\}) - \int_v k_{j1}(v)dv + (1-\delta)k_{j0}(v^s) - \sum_i \left[\overline{W}_{i0} n^r_{ij0}(v^s) + \kappa_i a_{ij0}\right]\mu_i \quad (103)$$

$$+\sum_{t=1}^{\infty} Q_{0,t} \sum_i \left(\begin{array}{c} \int_v \left\{\tilde{F}(k^a_{jt}(v), \{\mu_i n^a_{ijt}(v)\}) - \left[k_{jt+1}(v) - (1-\delta)k_{jt}(v)\right]\right\} dv \\ +\sum_i \left\{M_{ijt} \frac{u_{nit}}{u_{cit}} \left(\frac{n_{ijt}}{n_{it}}\right)^{\frac{1}{\omega}} + \mu_i \gamma_{ijt} \left[W_{ijt} - \frac{\mathcal{W}_{ijt-1}}{\lambda_w(\theta_{ijt-1})}\right] - [W_{ijt}\lambda_f(\theta_{ijt-1})a_{ijt-1} + \kappa_i a_{ijt}]\mu_i\right\} \end{array} \right),$$

subject to the constraints

$$\begin{aligned}
&\alpha^r_{ij0} : \mu_i n^r_{ij0}(v^s) \leq \mu_i n^s_{ij}(v^s) \\
&\alpha^a_{ij0} : \mu_i n^a_{ij0}(v^s) \leq \mu_i n^r_{ij}(v^s) \\
&Q_{0,1}\alpha_{ij1} : \mu_i n_{ij1} \leq (1-\sigma)\mu_i n^r_{ij0}(v^s) + \lambda_f(\theta_{ij0})\mu_i a_{ij0} \\
&\mu_i \chi^a_{ij0} : \ \frac{n^a_{ij0}(v^s)}{v^s_i} = (\leq) k^a_0(v^s) \\
&\eta^a_{k0} : k^a_0(v^s) \leq k_0(v^s) \\
&Q_{0,t+1}\alpha_{ijt+1} : \mu_i n_{ijt+1} \leq (1-\sigma)\mu_i n_{ijt} + \lambda_f(\theta_{ijt})\mu_i a_{ijt} \text{ for } t \geq 1 \\
&Q_{0,t}\mu_i \chi^a_{ijt}(v) : \ \frac{n^a_{ijt}(v)}{v_i} = (\leq) k^a_t(v) \text{ for } t \geq 1 \\
&Q_{0,t}\zeta^a_{ijt} : \int_v \mu_i n^a_{ijt}(v)dv \leq \mu_i n_{ijt} \text{ for } t \geq 1 \\
&Q_{0,t}\eta^a_{kt}(v) : k^a_t(v) \leq k_t(v) \text{ for } t \geq 1 \\
&\mu_i \eta_{ij0} : W_{ij0} \geq \overline{W}_{i0} = d_0 \overline{w}_i \\
&Q_{0,t+1}\mu_i \eta_{ijt+1} : W_{ijt+1} \geq \underline{W}_{t+1} = d_{t+1}\underline{w} \\
&Q_{0,t}\psi_{ijt} : \ a_{ijt} \geq 0 \\
&Q_{0,t}\gamma_{kjt}(v) : \ k_{jt+1}(v) \geq (1-\delta)k_{jt}(v),
\end{aligned}$$

where the inequalities in the above constraints indicate how the Lagrangian should be written so that the associated multipliers are positive. The FOCs of the firm's problem are

$$\begin{aligned}
&k^a_{j0}(v) : \ \tilde{F}_{kj0}(v^s) + \textstyle\sum_i \mu_i \chi^a_{ij0}(v^s) - \eta^a_{k0}(v) = 0 \\
&n^r_{ij0} : -\overline{W}_{i0} - \alpha^r_{ij0} + \alpha^a_{ij0} + Q_{0,1}(1-\sigma)\alpha^a_{ij1} = 0 \\
&n^a_{ij0} : \tilde{F}_{nij0}(v^s) - \alpha^a_{ij0} - \frac{\chi^a_{ij0}(v^s)}{v_i} = 0 \\
&a_{ij0} : -\kappa_i - Q_{0,1}W_{ij1}\lambda_f(\theta_{ij0}) + \lambda_f(\theta_{ij0})Q_{0,1}\alpha_{ij1} + \psi_{ij0} = 0 \\
&k^a_{jt}(v) : \ \tilde{F}_{kjt}(v) + \textstyle\sum_i \mu_i \chi^a_{ijt}(v) - \eta^a_{kt}(v) = 0 \\
&n^a_{ijt}(v) : \ \tilde{F}_{nijt}(v) - \frac{\chi^a_{ijt}(v)}{v_i} - \zeta^a_{ijt} = 0 \\
&a_{ijt} : \ -\kappa_i - Q_{t,t+1}W_{ijt+1}\lambda_f(\theta_{ijt}) + Q_{t,t+1}\alpha_{ijt+1}\lambda_f(\theta_{ijt}) + \psi_{ijt} = 0 \\
&k_{jt+1}(v) : \ -1 + Q_{t,t+1}(1-\delta) + Q_{t,t+1}\eta^a_{kt+1}(v) + \gamma_{kjt}(v) - Q_{t,t+1}\gamma_{kjt+1}(v)(1-\delta) = 0 \\
&\theta_{ijt} : W_{ijt+1}a_{ijt} = -\frac{\gamma_{ijt+1}}{\lambda_f(\theta_{ijt})} \frac{1-\eta}{\eta} (W_{ijt+1} + V_{ijt+1}) + \alpha_{ijt+1}a_{ijt} \\
&W_{ijt+1} : \ \gamma_{ijt+1} - \lambda_f(\theta_{ijt})a_{ijt} + \eta_{ijt+1} = 0
\end{aligned}$$

and

$$n_{ijt}:\left[\zeta^a_{ijt}+M_{ijt}\frac{u_{nit}}{u_{cit}}\frac{1}{\omega n_{ijt}}\left(\frac{n_{ijt}}{n_{it}}\right)^{\frac{1}{\omega}}\right]+(1-\sigma)Q_{t,t+1}\alpha_{ijt+1}-\alpha_{ijt}=0.$$

We now impose symmetry across firms j, which simplifies these FOCs to

$$\begin{aligned}
&k^a_0(v):\ \tilde{F}_{k0}(v^s)+\textstyle\sum_i\mu_i\chi^a_{i0}(v^s)-\eta^a_{k0}(v)=0\\
&n^a_{i0}(v^s):\ \tilde{F}_{ni0}(v^s)-\overline{W}_{i0}+(1-\sigma)Q_{0,1}\alpha_{ij1}-\chi^a_{i0}(v^s)/v_i=0\\
&a_{i0}:-\kappa_i-Q_{0,1}W_{ij1}\lambda_f(\theta_{i0})+\lambda_f(\theta_{i0})Q_{0,1}\alpha_{ij1}+\psi_{i0}=0\\
&k^a_t(v):\ \tilde{F}_{kt}(v)+\textstyle\sum_i\mu_i\chi^a_{it}(v)-\eta^a_{kt}(v)=0\\
&n^a_{it}(v):\ \tilde{F}_{nit}(v)-\chi^a_{it}(v)/v_i-\zeta^a_{it}=0\\
&a_{it}:\ -\kappa_i-Q_{t,t+1}W_{it+1}\lambda_f(\theta_{it})+Q_{t,t+1}\alpha_{it+1}\lambda_f(\theta_{it})+\psi_{it}=0\\
&k_{t+1}(v):\ -1+Q_{t,t+1}(1-\delta)+Q_{t,t+1}\eta^a_{kt+1}(v)+\gamma_{kt}(v)-Q_{t,t+1}\gamma_{kt+1}(v)(1-\delta)=0\\
&\theta_{it}:W_{it+1}a_{it}=-\frac{\gamma_{it+1}}{\lambda_f(\theta_{it})}\frac{1-\eta}{\eta}\left(W_{it+1}+V_{it+1}\right)+\alpha_{it+1}a_{it}\\
&W_{it+1}:\ \gamma_{it+1}-\lambda_f(\theta_{it})a_{it}+\eta_{it+1}=0
\end{aligned}$$

and

$$n_{it}:\left[\zeta^a_{it}+M_{it}\frac{u_{nit}}{u_{cit}}\frac{1}{\omega n_{it}}\right]+(1-\sigma)Q_{t,t+1}\alpha_{it+1}-\alpha_{it}=0.$$

This collection of FOCs and constraints fully characterizes the firm problem.

C.2 Restricted Disaggregate and Restricted Aggregate Firm Problems

The solution to the original firm problem characterized above is difficult to compute because a firm can potentially operate an infinite number of capital types v. Next, we show that under the assumption that firms fully utilize all capital types, we can summarize this distribution of types using only few aggregate state variables, that is, aggregate output and the aggregate employment of each worker type. Specifically, we start from the original firm problem (103) and impose the additional constraint that the firm must always fully utilize all types of capital. Because we started the problem at the original steady state without the minimum wage, the Leontief structure and the restriction that a firm must operate all of its capital imply that the firm cannot fire any workers at $t=0$. We begin with the following lemma.

Lemma 4. *If $\sigma>\delta$, then $a_{ijt}>0$ for all i, j, and t in the restricted disaggregate firm problem.*

Proof: Since the CES production function has an Inada condition in each labor type, we know that in the original steady state a strictly positive amount of each type of labor is employed to operate $k(v^s)$. Since we assume that along the transition all capital is utilized and that $\sigma>\delta$, the firm must continue to hire a strictly positive amount of labor a_{it} for all i in all periods, so $\psi_{it}=0$. Hence, the nonnegativity constraint on a_{ijt} never binds, and we can drop this constraint. ■

Given this result, we can reformulate the restricted disaggregate firm problem as

$$\max\tilde{F}(k_{j0}(v^s),\{\mu_i n^s_{ij0}(v^s)\})-\textstyle\int_v k_{j1}(v)dv+(1-\delta)k_{j0}(v^s)-\sum_i\left[\overline{W}_{i0}n^s_{ij0}(v^s)+\kappa_i a_{ij0}\right]\mu_i \quad (104)$$

$$+\sum_{t=1}^{\infty} Q_{0,t}\sum_i \left\{\int_v \left\{\tilde{F}(k_{jt}(v),\{\mu_i n_{ijt}(v)\}) - [k_{jt+1}(v)-(1-\delta)k_{jt}(v)]\right\} dv\right\}$$

$$+\sum_{t=1}^{\infty} Q_{0,t}\sum_i \left\{M_{ijt}\frac{u_{nit}}{u_{cit}}\left(\frac{n_{ijt}}{n_{it}}\right)^{\frac{1}{\omega}} + \mu_i\gamma_{ijt}\left[W_{ijt} - \frac{\mathcal{W}_{ijt-1}}{\lambda_w(\theta_{ijt-1})}\right] - [W_{ijt}\lambda_f(\theta_{ijt-1})a_{ijt-1} + \kappa_i a_{ijt}]\mu_i\right\},$$

subject to

$$\begin{aligned}
&Q_{0,1}\alpha_{ij1} : \mu_i n_{ij1} \leq (1-\sigma)\mu_i n^s_{ij0}(v^s) + \lambda_f(\theta_{ij0})\mu_i a_{ij0}\\
&Q_{0,t+1}\alpha_{ijt+1} : \mu_i n_{ijt+1} \leq (1-\sigma)\mu_i n_{ijt} + \lambda_f(\theta_{ijt})\mu_i a_{ijt} \text{ for } t \geq 1\\
&Q_{0,t}\mu_i\chi_{ijt}(v) : \ \frac{n_{ijt}(v)}{v_i} = (\leq) k_t(v) \text{ for } t \geq 1\\
&Q_{0,t}\zeta_{ijt} : \int_v \mu_i n_{ijt}(v)dv \leq \mu_i n_{ijt} \text{ for } t \geq 1\\
&\mu_i\eta_{ij0} : W_{ij0} \geq \overline{W}_{i0} = d_0\overline{w}_i\\
&Q_{0,t+1}\mu_i\eta_{ijt+1} : W_{ijt+1} \geq \underline{W}_{t+1} = d_{t+1}\underline{w}\\
&Q_{0,t}\gamma_{kjt}(v) : \ k_{jt+1}(v) \geq (1-\delta)k_{jt}(v).
\end{aligned}$$

As before, the firm chooses sequences a_{ij0}, $k_{jt}(v)$, $\{n_{ijt}(v)\}$, a_{ijt}, $k_{jt+1}(v)$, θ_{ijt}, W_{ijt+1}, and n_{ijt+1}. The FOCs associated with these choices are

$$\begin{aligned}
&n_{ijt}(v) : \tilde{F}_{nijt}(v) - \frac{\chi_{ijt}(v)}{v_i} - \zeta_{ijt} = 0 \text{ for } t \geq 1\\
&a_{ijt} : \ -\kappa_i - Q_{t,t+1}W_{ijt+1}\lambda_f(\theta_{ijt}) + Q_{t,t+1}\alpha_{ijt+1}\lambda_f(\theta_{ijt}) = 0\\
&k_{jt+1}(v) : Q_{t,t+1}\left[\tilde{F}_{kjt+1}(v) + \sum_i \mu_i\chi_{ijt+1}(v)\right] - 1 + Q_{t,t+1}(1-\delta) + \gamma_{kjt}(v) - Q_{t,t+1}\gamma_{kjt+1}(v)(1-\delta) = 0\\
&\theta_{ijt} : W_{ijt+1}a_{ijt} = -\frac{\gamma_{ijt+1}}{\lambda_f(\theta_{ijt})}\frac{1-\eta}{\eta}(W_{ijt+1} + V_{ijt+1}) + \alpha_{ijt+1}a_{ijt}\\
&W_{ijt+1} : \ \gamma_{ijt+1} - \lambda_f(\theta_{ijt})a_{ijt} + \eta_{ijt+1} = 0\\
&n_{ijt} : \left[\zeta_{ijt} + M_{ijt}\frac{u_{nit}}{u_{cit}}\frac{1}{\omega n_{ijt}}\left(\frac{n_{ijt}}{n_{it}}\right)^{\frac{1}{\omega}}\right] + (1-\sigma)Q_{t,t+1}\alpha_{ijt+1} - \alpha_{ijt} = 0.
\end{aligned}$$

Imposing symmetry across j then gives the following simpler version of the above system

$$\begin{aligned}
&n_{it}(v) : \ \tilde{F}_{nit}(v) - \frac{\chi_{it}(v)}{v_i} - \zeta_{it} = 0\\
&a_{it} : \ -\kappa_i - Q_{t,t+1}W_{it+1}\lambda_f(\theta_{it}) + Q_{t,t+1}\alpha_{it+1}\lambda_f(\theta_{it}) = 0\\
&k_{t+1}(v) : Q_{t,t+1}\left[\tilde{F}_{kt+1}(v) + \sum_i \mu_i\chi_{it+1}(v)\right] - 1 + Q_{t,t+1}(1-\delta) + \gamma_{kt}(v) - Q_{t,t+1}\gamma_{kt+1}(v)(1-\delta) = 0\\
&\theta_{it} : W_{it+1}a_{it} = -\frac{\gamma_{it+1}}{\lambda_f(\theta_{it})}\frac{1-\eta}{\eta}(W_{it+1} + V_{it+1}) + \alpha_{it+1}a_{it}\\
&W_{it+1} : \ \gamma_{it+1} - \lambda_f(\theta_{it})a_{it} + \eta_{it+1} = 0\\
&n_{it} : \left[\zeta_{it} + M_{it}\frac{u_{nit}}{u_{cit}}\frac{1}{\omega n_{it}}\right] + (1-\sigma)Q_{t,t+1}\alpha_{it+1} - \alpha_{it} = 0.
\end{aligned}$$

This collection of FOCs and constraints fully characterizes the symmetric equilibrium solution.

Restricted Aggregate Firm Problem. We now show that the solution to this restricted problem can be simplified by replacing the distribution of capital types with just few aggregate variables as states, namely, aggregate output and the aggregate employment of each worker type.

To do so, start from the restricted disaggregate firm problem (104). Since, by assumption, all capital is fully utilized and $k_{jt}(v) = n_{ijt}(v)/v_i$,

$$y_{jt} \equiv \int_v \min\left\{k_{jt}(v), \left\{\frac{n_{ijt}(v)}{v_i}\right\}\right\} f(v)dv = \int_v k_{jt}(v)f(v)dv. \tag{105}$$

Next, we construct the sufficient aggregate state variables y_{jt} and n_{ijt}. To construct y_{jt}, we start with the capital accumulation law and do some manipulations to end up with a law of motion on aggregate output as follows:

$$k_{jt+1}(v) = (1-\delta)k_{jt}(v) + x_t(v) \tag{106}$$
$$k_{jt+1}(v)f(v) = (1-\delta)k_{jt}(v)f(v) + x_t(v)f(v) \tag{107}$$
$$\int_v k_{jt+1}(v)f(v)dv = (1-\delta)\int_v k_{jt}(v)f(v)dv + \int_v x_t(v)f(v)dv \tag{108}$$
$$y_{jt+1} = (1-\delta)y_{jt} + \int_v x_{jt}(v)f(v)dv. \tag{109}$$

Here, we have multiplied the capital accumulation law (106) by $f(v)$ in (107), integrated over v in (108), and then used the assumption $k_{jt}(v) = n_{ijt}(v)/v_i$ to arrive at (109).

Next, we construct the law of motion for the aggregate measures of workers of each type n_{ijt+1} from the allocations of such workers to each type of capital $n_{ijt}(v)$. To do so, we again start from the capital accumulation equation and do some manipulations as follows:

$$\frac{n_{ijt+1}(v)}{v_i} = (1-\delta)\frac{n_{ijt}(v)}{v_i} + x_{jt}(v) \tag{110}$$
$$n_{ijt+1}(v) = (1-\delta)n_{ijt}(v) + v_i x_{jt}(v) \tag{111}$$
$$\int_v n_{ijt+1}(v)dv = (1-\delta)\int_v n_{ijt}(v)dv + \int_v v_i x_{jt}(v)dv \tag{112}$$
$$n_{ijt+1} = (1-\delta)n_{ijt} + \int_v v_i x_{jt}(v)dv. \tag{113}$$

In (110), we substitute $k_t(v) = n_t(v)/v$ into the capital accumulation law (106), multiply by v_i in (111), integrate over v in (112), and then obtain (113).

Given these aggregate variables, we define the restricted aggregate firm's problem as follows.

$$\max y_{j0} - \int_v x_{j0}(v)dv - \sum_i \left[\overline{W}_{i0} n^s_{ij0}(v^s) + \kappa_i a_{ij0}\right]\mu_i \tag{114}$$
$$+\sum_{t=1}^{\infty} Q_{0,t}\sum_i \left(y_{jt} - \int_v x_{jt}(v)dv + \left\{M_{ijt}\frac{u_{nit}}{u_{cit}}\left(\frac{n_{ijt}}{n_{it}}\right)^{\frac{1}{\omega}} + \mu_i\gamma_{ijt}\left[W_{ijt} - \frac{\mathcal{W}_{ijt-1}}{\lambda_w(\theta_{ijt-1})}\right]\right\}\right)$$
$$-\sum_{t=1}^{\infty} Q_{0,t}\sum_i [W_{ijt}\lambda_f(\theta_{ijt-1})a_{ijt-1} + \kappa_i a_{ijt}]\mu_i,$$

subject to

$$Q_{0,1}\alpha_{ij1} : \mu_i n_{ij1} \leq (1-\sigma)\mu_i n^s_{ij0}(v^s) + \lambda_f(\theta_{ij0})\mu_i a_{ij0}$$
$$Q_{0,t+1}\alpha_{ijt+1} : \mu_i n_{ijt+1} \leq (1-\sigma)\mu_i n_{ijt} + \lambda_f(\theta_{ijt})\mu_i a_{ijt} \text{ all } t \geq 1$$
$$Q_{0,t}\lambda^y_{jt} : y_{jt+1} = (\leq)(1-\delta)y_{jt} + \int_v x_{jt}(v)f(v)dv$$
$$Q_{0,t}\lambda^n_{ijt} : \mu_i n_{ijt+1} = (\leq)(1-\delta)\mu_i n_{ijt} + \int_v v_i x_{jt}(v)dv$$
$$\mu_i\eta_{ij0} : W_{ij0} \geq \overline{W}_{i0} = d_0\overline{w}_i$$
$$Q_{0,t+1}\mu_i\eta_{ijt+1} : W_{ijt+1} \geq \underline{W}_{t+1} = d_{t+1}\underline{w}$$
$$Q_{0,t}\gamma_{kjt}(v) : \; x_{jt}(v) \geq 0.$$

Notice that for this problem (114), we do not record the distribution of capital stocks as a state. The restricted aggregate problem gives the allocations for $(a_{ijt}, n_{ijt+1}, y_{jt}, W_{ijt+1}, \theta_{ijt})$ and the disaggregate investment choices $\{x_{jt}(v)\}$. The next lemma shows that in the solution to this problem, the firm invests in a single type of capital in each period.

***Lemma* 5.** *In the restricted aggregate firm problem in (114), a firm invests in at most one type of capital per period, that is, for each t, there is at most one v_t such that $x_{jt}(v_t) > 0$.*

Proof: The FOC for $x_{jt}(v)$ is $\gamma_{kjt}(v) = 1 - \lambda^y_{jt} f(v) - \sum_i v_i \lambda^n_{ijt}$. Since $f(v)$ is concave, $-f(v)$ is convex, so $\gamma_{kjt}(v)$ is convex. Thus, it has a unique global minimum. Thus, exist a unique v^*_t for each t such that $\gamma_{kjt}(v) = 0$. Thus, $x_{jt}(v) = 0$ for all $v \neq v^*_t$, where v^*_t is given by $\lambda^y_{jt} f_i(v^*_t) = -\lambda^n_{ijt}$. ■

Next, we want to argue that a solution to the restricted aggregate problem can be used to construct a *unique* candidate solution to the restricted disaggregate problem. But when we do so, we note that the restricted disaggregate problem includes $\{n_{ijt}(v)\}$, namely, the allocation of workers to each type of capital v along with the vector of capital of each type $\{k_{ijt+1}(v)\}$, as well as the aggregate variables $\{n_{ijt}, a_{ijt}, \theta_{ijt}, W_{ijt}\}$. So at first blush, it seems that if we simply use the adding up constraints on capital and labor $\int_v n_{ijt}(v)dv = n_{ijt}$ and $\int_v k_{ijt+1}(v)dv = k_{ijt+1}$, we do not have enough information to pin down the disaggregate part of the allocation. But if we recall that the entire construction was made under the presumption that in the disaggregate problem all capital was fully utilized in that

$$n_{ijt}(v) = v_i k_t(v) \tag{115}$$

for all i and $t \geq 1$ and v, then we can.

To do so formally, let $\{\hat{n}_{ijt}, \hat{y}_{jt}, \hat{x}_{jt}(v^*_t), \hat{W}_{ijt}\}$ be the allocations and wages that solve the restricted aggregate problem. We construct the associated allocations and wages for the restricted disaggregate problem, $\{n^*_{ijt}(v), k^*_{ijt+1}(v)\}$ along with $\{n^*_{ijt}, y^*_{jt}, k^*_{jt+1}, W^*_{ijt}\}$ as follows. Obviously, we define the *aggregate component* of the disaggregate problem to be equal to that of the aggregate problem by defining $(n^*_{ijt}, y^*_{jt}, k^*_{jt+1}, W^*_{ijt})$ for each t by

$$(n^*_{ijt}, y^*_{jt}, k^*_{jt+1}, W^*_{ijt}) = (\hat{n}_{ijt}, \hat{y}_{jt}, \hat{x}_{jt}(v^*_t), \hat{W}_{ijt}).$$

More interestingly, we construct the disaggregate vector of capital stocks for each type as follows. For notational simplicity only, we begin by presuming that, as we found in our quantitative exercise, that the type of capital v^*_t from any period t was never invested again. Formally, among all the capital types $\{v^s, v^*_0, v^*_1, \dots\}$ it turned out that $v^*_t \neq v^*_r$ for all t, r and $v^*_t \neq v^s$ for all t. Under this presumption, any capital or investment that was made is simply allowed to depreciate away. Hence, the vector of capital stocks at t is

$$k_t(v^s) = (1-\delta)^t k_0(v^s),\ k_t(v^*_1) = (1-\delta)^{t-1} x_t(v^*_1), \dots, k_t(v^*_{t-2}) = (1-\delta) x_t(v^*_{t-2}),\ k_t(v^*_{t-1}) = x_t(v^*_{t-1}),$$

where we have simply used that the law of motion for capital of any type v is

$$k_{t+1}(v) = (1-\delta)k_t(v) + x_t(v)$$

and that $k_t(v^*_{t-s}) = 0$ for all $s \geq 1$. Now, if instead the same type of capital is invested in more than once, we just add that in the appropriate way. For example, if $v^*_1 = v^*_3$ then $k_2(v^*_1) = x_1(v^*_1)$, $k_3(v^*_1) = (1-\delta)x_1(v^*_1)$, and $k_4(v^*_1) = (1-\delta)^2 x_1(v^*_1) + x_3(v^*_3)$ and collapse the types of capital by one by identifying v^*_1 and v^*_3 as $v^*_{1,3}$.

Next, we can construct the disaggregate allocations of labor using these constructed capital series using (115) as follows

$$n^*_{ijt}(v^*_r) = v^*_i k_t(v^*_r) \text{ for all } r \in \{v^s, v^*_0, v^*_1, \ldots\}.$$

Thus, we have a unique mapping from the restricted aggregate allocations $\{\hat{n}_{ijt}, \hat{y}_{jt}, \hat{x}_{jt}(v^*_t), \hat{W}_{ijt}\}$ to the restricted disaggregate allocations $\{n^*_{ijt}(v), k^*_{ijt+1}(v)\}$ along with $\{n^*_{ijt}, y^*_{jt}, k^*_{jt+1}, W^*_{ijt}\}$. Thus we have used the solution to the restricted aggregate problem to construct a *candidate solution to the restricted disaggregate problem.* We then have the following lemma:

***Lemma* 6.** *Under the assumption that it is optimal to fully utilize capital in every period, the candidate solution to the restricted disaggregate firm problem constructed from the solution to the restricted aggregate firm problem solves the restricted disaggregate firm problem.*

Proof: The restricted disaggregate problem and the restricted aggregate problem are both convex problems. Hence, both have unique solutions. By construction of the restricted aggregate problem, the solution to the restricted disaggregate problem solves the restricted aggregate problem. Thus, the solutions to the two problems coincide. ■

Consider now verifying that our candidate solution is a solution to the original problem. To do so, we proceed as follows. Once we solve the restricted aggregate problem, we then use the mapping just discussed to construct a unique candidate solution for the original problem. Then all we need to do is to construct candidate multipliers for the original problem by evaluating the first-order conditions of the original problem at our candidate solution derived from the restricted disaggregate problem. If those multipliers are consistent with it being optimal for the firm to fully utilize capital of each type in each period and the total labor supply then our candidate solution is the unique solution to the original problem. We summarize this discussion in the following lemma.

***Lemma* 7.** *When we evaluate the symmetric first-order conditions to the original problem at our candidate solution derived from the restricted aggregate firm problem, if the multipliers α^r_{i0} on the constraints for retained workers in $t = 0$ and the multipliers α^r_{i0} on the constraints for active workers in $t = 0$ are nonnegative for each i, the multipliers ζ^a_{it} on the constraints for active workers in $t \geq 1$ are positive for each i, and the multipliers $\eta^a_{kt}(v)$ on the constraints for the full usage of capital of each type $v \in \{v^s, v^*_0, v^*_1, \ldots\}$ for which a strictly positive amount is used in t are positive, then in the original firm problem all capital is always utilized and no worker is fired.*

In sum, we only ever solve the restricted aggregate problem, which is a relative simple one with few aggregate state variables. We then use the solution to this problem to construct a candidate solution to the original problem and verify that at this candidate solution, the multipliers for the original problem have the correct signs, namely, the signs consistent with our assumption of full capital utilization.