Micro Data for Macro Models

Topic 2: Capital Investment and Adjustment Costs

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Plan for this Topic

1. An unfair summary of the empirical investment literature

2. Accounting for micro-level investment behavior with nonconvex adjustment costs

3. Macro implications of nonconvex adjustment costs

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1. **An unfair summary of the empirical investment literature**

2. Accounting for micro-level investment behavior with nonconvex adjustment costs

3. Macro implications of nonconvex adjustment costs

Empirical Investment Literature

> *The empirical investment literature is full of disappointments. From time to time waves of new ideas challenge the aggregate investment equation, but these challenges are rarely successful, and progress is, at best, slow. There are serious theoretical obstacles, stemming mostly from the richness of the cross-sectional and time-series scenarios faced by actual investors, from the complexity of the investment technologies available to them, and from the myriad incentive problems that these agents face. There are at least as complex, and perhaps insurmountable, data problems. Both right- and left-hand side variables are seldom measured properly.*

Caballero, Engel, and Haltiwanger, “Plant-Level Adjustment and Aggregate Investment Dynamics”

Empirical Investment Literature

- Many early papers focus on neoclassical model
  - "User cost" and "q theory" formulations
  - Finds model does not fit the data well at micro or macro level

- Two main responses:
  1. Real adjustment frictions with nonconvexities
  2. Financial frictions to acquiring investment funds are important

The Neoclassical Model

Consider individual firm investment problem:

- Firm i with production function

$$y_{it} = k_{it}^{\alpha}, \alpha \leq 1$$

- Invest to accumulate capital $k_{it+1} = (1-\delta)k_{it} + i_{it}$
- Quadratic adjustment costs $-\frac{\phi}{2}\left(\frac{i_{it}}{k_{it}}\right)^2 k_{it}$
- Discount future at constant rate r

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$$v(k_{it}) = \max_{i_{it}, k_{it+1}} k_{it}^{\alpha} - i_{it} - \frac{\phi}{2}\left(\frac{i_{it}}{k_{it}}\right)^2 k_{it} + \frac{1}{1+r} v(k_{it+1})$$
$$\text{such that } k_{it+1} = (1-\delta)k_{it} + i_{it}$$

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$$\text{such that } k_{it+1} = (1-\delta)k_{it} + i_{it} \qquad (\times q_{it})$$

Take **first order conditions**:

$$\begin{aligned} 1 + \phi(\frac{i_{it}}{k_{it}}) =& q_{it} \\ q_{it} =& v'(k_{it+1}) \\ =& \frac{1}{1+r}\sum_{s=0}^{\infty}\left(\frac{1-\delta}{1+r}\right)^s \left(\alpha k_{it+s+1}^{\alpha-1} + \Phi_{it+s+1}\right) \end{aligned}$$

The User Cost Model: $\phi = 0$

With $\phi = 0$, first order conditions simplify to

$$q_{it} = 1$$

$$\underbrace{\alpha k_{it+1}^{\alpha-1}}_{\text{MPK}_{it}} = \underbrace{r + \delta}_{\text{user cost}}$$

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\end{aligned}
$$

- The **user cost of capital** is the implicit rental rate on capital
- Typically extended to incorporate other empirically relevant features:

$$
u_{st} = \underbrace{p_t}_{\text{relative price of capital}} \times \underbrace{\frac{1 - m_{st} - z_{st}}{1 - \tau_t}}_{\text{taxes}} \times (r_t + \delta_s)
$$

Empirical Performance of the User Cost Model

- Typical regression takes the form

$$\frac{i_{it}}{k_{it}} = \alpha_i + \beta u_{it} + \Gamma \text{other variables}_{it} + \varepsilon_{it}$$

- Two main failures of user cost model:
    1. Estimated user cost elasticity β small ($\approx$ 0 to -0.5)
    2. Coefficients on other variables, especially cash flow, large and significant

- Hall and Jorgensen (1967); Cummins, Hassett, and Hubbard (1994); Chirinko, Fazarri, and Meyer (1999)

The Q-Theory Model: $\phi \geq 0$

$$
\begin{aligned}
q_{it} =& 1 + \phi(\frac{i_{it}}{k_{it}}) \\
q_{it} =& v'(k_{it+1}) = \frac{1}{1+r} \sum_{s=0}^{\infty} \left(\frac{1-\delta}{1+r} \right)^{s} \left(\alpha k_{it+s+1}^{\alpha-1} + \Phi_{it+s+1} \right)
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- Two key implications of the model:
  1. q_{it} is the marginal value of capital to the firm
  2. Investment positively related to q_{it}: $\frac{i_{it}}{k_{it}} = \frac{1}{\phi}(q_{it} - 1)$

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  2. Investment positively related to q_{it}: $\frac{i_{it}}{k_{it}} = \frac{1}{\phi}(q_{it} - 1)$
- Hayashi (1982): under constant returns, $v'(k_{it}) = \frac{v(k_{it})}{k_{it}}$
  - Marginal q = average q (sometimes called Tobin's q)
  - Extend to include relative price, taxes, etc.

Empirical Performance of the Q Model

- Typical regression takes the form

$$\frac{i_{it}}{k_{it}} = \alpha_i + \beta q_{it} + \Gamma \text{other variables}_{it} + \varepsilon_{it}$$

- Two main failures of the Q model:
    1. Estimated coefficient β small and unstable
    2. Coefficients on other variables, especially cash flow, large and significant

- Summers (1981); Cummins, Hassett, and Hubbard (1994); Erickson and Whited (2000)

Doms and Dunne (1998)

Two responses to failure of neoclassical model:

1. Nonconvex adjustment costs are important
2. Financial frictions to acquiring investment funds are important

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Doms and Dunne (1998):

- Landmark descriptive study of investment in LRD
- Shows micro-level investment is lumpy, i.e., occurs mainly along extensive margin
    - Fluctuations in total investment mainly due to extensive margin
- Suggests important role for fixed adjustment costs

Measurement

- Use Census data from LRD, 1972 - 1988
    - After 1988, stopped collecting book value of capital
- Construct capital stock using perpetual inventory method
    - Focus on balanced panel
- Analyze the growth rate of capital for plant *i* at time *t*

$$GK_{it} = \frac{i_{it} - \delta k_{it-1}}{0.5 \times (k_{it-1} + k_{it})}$$

Plant-Level Investment is Lumpy Across Plants

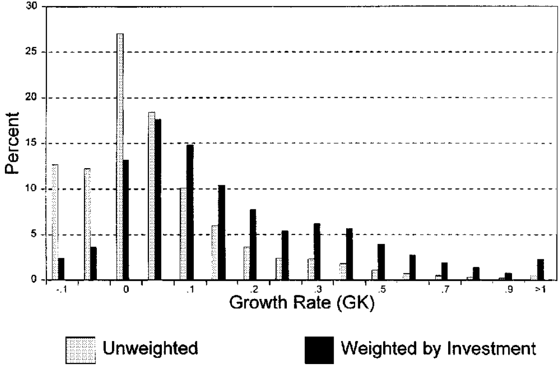

FIG. 1. Capital growth rate (GK) distributions: Unweighted and weighted by investment.

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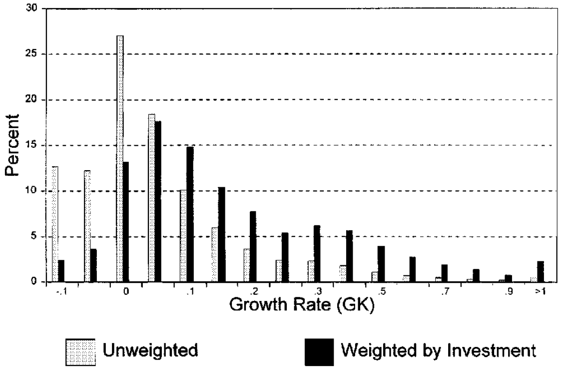
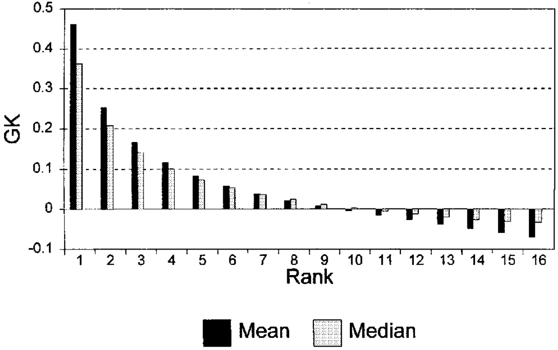


FIG. 1. Capital growth rate (GK) distributions: Unweighted and weighted by investment.

- 51.9% of plants increase capital $\leq 2.5\%$
- 11% of plants increase capital $\geq 20\%$

Plant-Level Investment is Lumpy Within Plants



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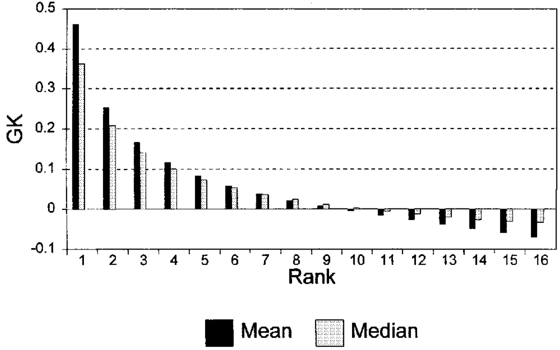

- Capital growth in largest investment episode nearly 50%
- In median investment episode approximately 0%

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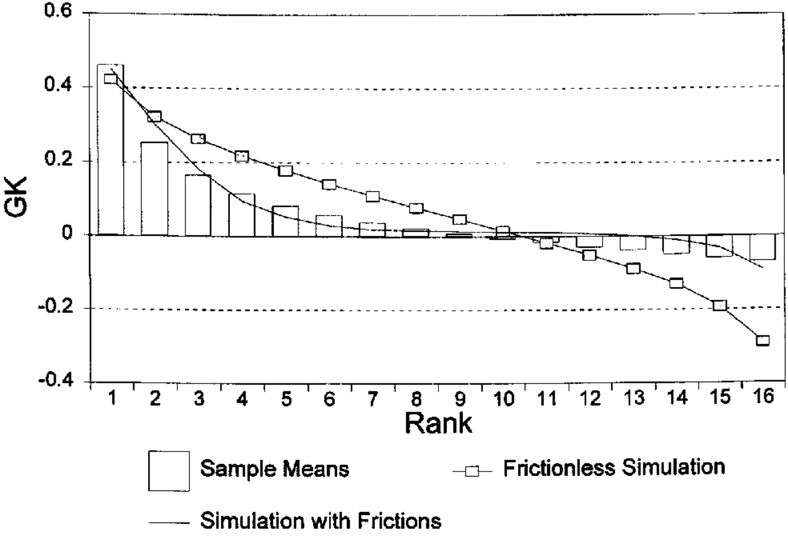
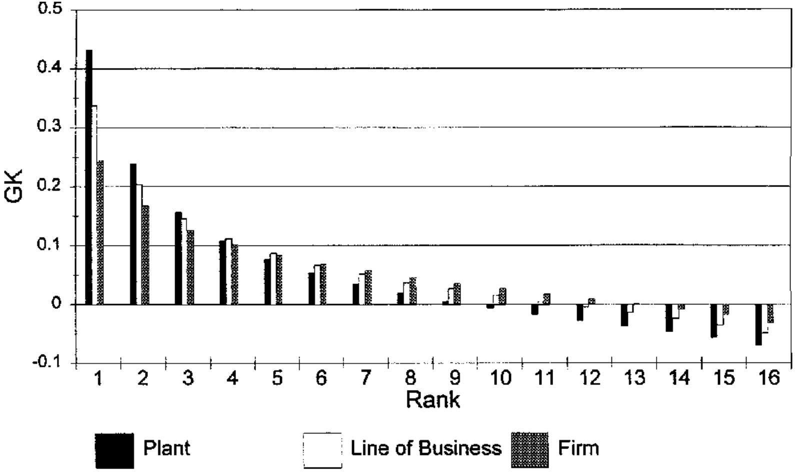
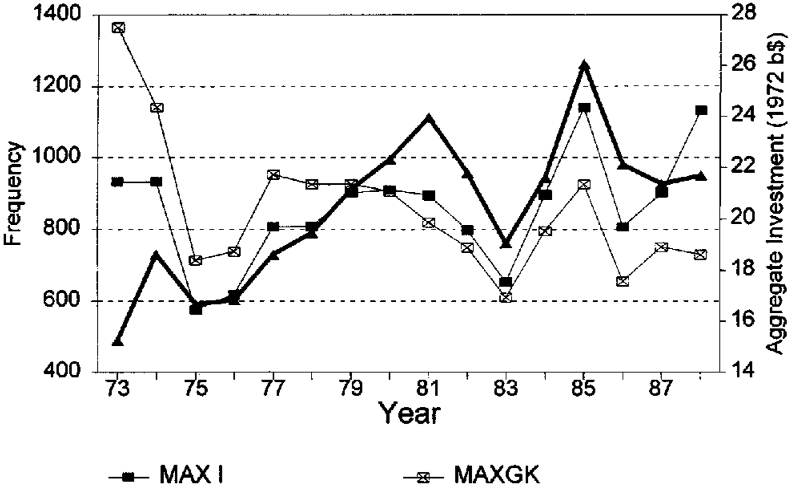


Plant-Level Investment Lumpier than Firm-Level



Frequency of Spikes Correlated with Aggregate Investment



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Zwick and Mahon (2016):

- Clean study exploiting exploiting policy-induced variation in cost of capital
- Shows investment very responsive to cost, especially for small/non-dividend paying firms
- Suggests important role for financial frictions (and potentially fixed costs)

Bonus Depreciation Allowance

Table 1: Regular and Bonus Depreciation Schedules for Five Year Items

Normal Depreciation							
Year	0	1	2	3	4	5	Total
Deductions (000s)	200	320	192	115	115	58	1000
Tax Benefit ($\tau = 35\%$)	70	112	67.2	40.3	40.3	20.2	350
Bonus Depreciation (50%)							
Year	0	1	2	3	4	5	Total
Deductions (000s)	600	160	96	57.5	57.5	29	1000
Tax Benefit ($\tau = 35\%$)	210	56	33.6	20.2	20.2	10	350

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- Bonus shifts depreciation allowances from future to present
- With discounting, lowers the total cost of investment

  $\Longrightarrow$ Bonus more valuable for longer-lived investment

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$$z_{s0} = \sum_{t=0}^{T_s} \frac{1}{(1+r)^t} D_t$$

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$$z_{st} = \theta \times 1 + (1-\theta) \times z_{s0}$$

Data

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  - Released by Statistics of Income division of IRS
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- Investment i_{it} measured as expenditures on equipment eligible for Bonus
- PV of depreciation allowances z_{st} constructed at four digit level using $r = 7\%$

Identification Strategy

- Identify effect of policy using difference-in-differences design
  - Treatment group: firms in long-lived industries
  - Control group: firms in short-lived industries

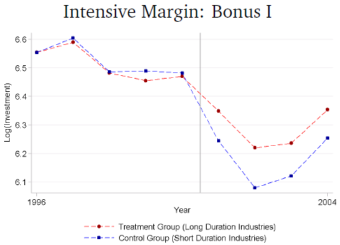
- Regression specification

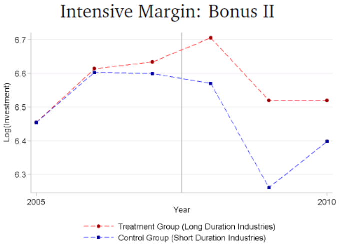
$$f(i_{it}, k_{it}) = \alpha_i + \delta_t + \beta g(z_{st}) + \gamma X_{it} + \varepsilon_{it}$$

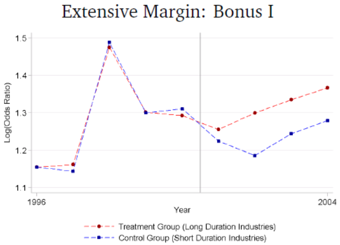
  - $f(i_{it}, k_{it})$: $\log i_{it}$, $\log \frac{p_{st}}{1-p_{st}}$, or $\frac{i_{it}}{k_{it}}$
  - $g(z_{st})$: z_{st} or $\frac{1-\tau z_{st}}{1-\tau}$

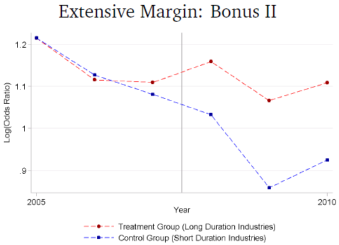
- Key assumption for difference-in-differences: parallel trends holds

Graphical Evidence









Overall Effect of Bonus on Investment

	Intensive Margin: LHS Variable is log(Investment)					
	(1)	(2)	(3)	(4)	(5)	(6)
$z_{N,t}$	3.69***	3.78***	3.07***	3.02***	3.73***	4.69***
	(0.53)	(0.57)	(0.69)	(0.81)	(0.70)	(0.62)
$CF_{it}/K_{i,t-1}$		0.44***				
		(0.016)				
Observations	735341	580422	514035	221306	585914	722262
Clusters (Firms)	128001	100883	109678	63699	107985	124962
R^2	0.71	0.74	0.73	0.80	0.72	0.71
	Extensive Margin: LHS Variable is log(P(Investment > 0))					
	(1)	(2)	(3)	(4)	(5)	(6)
$z_{N,t}$	3.79**	3.87**	3.12	3.59**	3.99*	4.00***
	(1.24)	(1.21)	(2.00)	(1.14)	(1.69)	(1.13)
$CF_{it}/K_{i,t-1}$		0.029**				
		(0.0100)				
Observations	803659	641173	556011	247648	643913	803659
Clusters (Industries)	314	314	314	274	277	314
R^2	0.87	0.88	0.88	0.93	0.90	0.90
	Tax Term: LHS Variable is Investment/Lagged Capital					
	(1)	(2)	(3)	(4)	(5)	(6)
$\frac{1-t_c z}{1-t_c}$	-1.60***	-1.53***	-2.00***	-1.42***	-2.27***	-1.50***
	(0.096)	(0.095)	(0.16)	(0.13)	(0.14)	(0.10)
$CF_{it}/K_{i,t-1}$		0.043***				
		(0.0023)				
Observations	637243	633598	426214	211029	510653	631295
Clusters (Firms)	103890	103220	87939	57343	90145	103565
R^2	0.43	0.43	0.48	0.54	0.45	0.44
Controls	No	No	No	No	Yes	No
Industry Trends	No	No	No	No	No	Yes

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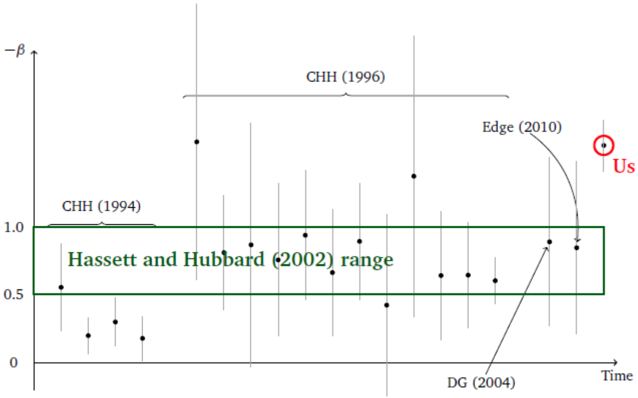
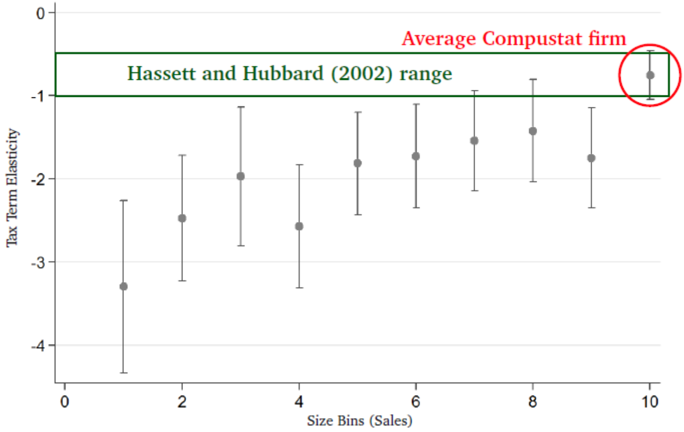
Larger Effect Than Existing Literature



Heterogeneity Suggestive of Financial Frictions

	Sales		Div Payer?		Lagged Cash	
	Small	Big	No	Yes	Low	High
$z_{N,t}$	6.29***	3.22***	5.98***	3.67***	7.21***	2.76**
	(1.21)	(0.76)	(0.88)	(0.97)	(1.38)	(0.88)
Equality Test	$p = .030$		$p = .079$		$p = .000$	
Observations	177620	255266	274809	127523	176893	180933
Clusters (Firms)	29618	29637	39195	12543	45824	48936
R^2	0.44	0.76	0.69	0.80	0.81	0.76

Heterogeneity Explains Larger Estimate than Literature



Unfair Review of Empirical Investment Lit

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- Two responses to failure of neoclassical model
  1. Adjustment costs feature nonconvexities
  2. Financial frictions influence investment behavior

The Rest of This Topic

Focus on role of nonconvex adjustment costs in explaining micro and macro investment dynamics

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Focus on role of nonconvex adjustment costs in explaining micro and macro investment dynamics

1. **Models of micro-level investment behavior**

2. **Aggregate implications of these models**
    - Aggregation
    - General equilibrium

Plan for this Topic

1. An unfair summary of the empirical investment literature

2. **Accounting for micro-level investment behavior with nonconvex adjustment costs**

3. Macro implications of nonconvex adjustment costs

Cooper and Haltiwanger (2006)

- What types of adjustment costs do we need to match micro-level investment behavior?
  - Pays special attention to lumpy nature of investment

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  - Does NOT mean equilibrium in one market!

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LRD Data

Sample

- Establishment-level observations
- Balanced panel: model abstracts from entry and exit
- 1972 - 1988: want to use data on expenditures and retirements

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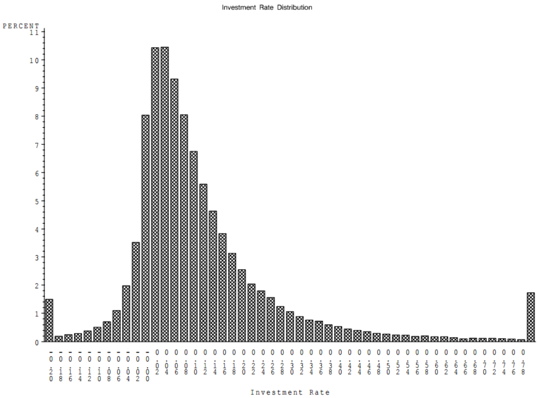
Measurement

- Investment i_{it}: $\text{expenditure}_{it} - \text{retirments}_{it}$
- Capital k_{it}: $k_{it+1} = (1 - \delta_{it})k_{it} + i_{it}$
- Depreciation δ_{it}: constructed to reflect in-use depreciation

Cross-Sectional Distribution of Investment Rates



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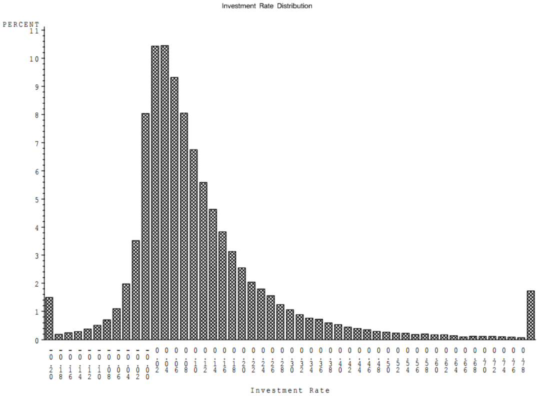

- Large mass of observations near zero
- Highly skewed and fat right tails

Cross-Sectional Distribution of Investment Rates

Variable	LRD
Average Investment Rate	12.2% (0.10)
Inaction Rate: Investment	8.1% (0.08)
Fraction of Observations with Negative Investment	10.4% (0.09)
Spike Rate: Positive Investment	18.6% (0.12)
Spike Rate: Negative Investment	1.8% (0.04)
Serial correlation of Investment Rates	0.058 (0.003)
Correlation of Profit Shocks and Investment	0.143 (0.003)

General Investment Model

Bellman equation

$$v(z_{it}, k_{it}) = \max_{i_{it}} e^{z_{it}} k_{it}^{\alpha} - p(i_{it}) i_{it} - c(i_{it}, k_{it}, z_{it})$$
$$+ \frac{1}{1+r} \mathbb{E}_t [v(z_{it+1}, (1-\delta) k_{it} + i_{it})]$$

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& + \frac{1}{1+r} \mathbb{E}_t [v(z_{it+1}, (1-\delta) k_{it} + i_{it})]
\end{aligned}
$$

Adjustment costs

$$
c(i_{it}, k_{it}, z_{it}) = \underbrace{\frac{\gamma}{2} \left(\frac{i_{it}}{k_{it}} \right)^2 k_{it}}_{\text{convex}} + \underbrace{\mathbb{1}\left(i_{it} \neq 0\right) \left(F k_{it} + \lambda e^{z_{it}} k_{it}^{\alpha}\right)}_{\text{nonconvex}}
$$

General Investment Model

Bellman equation

$$v(z_{it}, k_{it}) = \max_{i_{it}} e^{z_{it}} k_{it}^{\alpha} - p(i_{it}) i_{it} - c(i_{it}, k_{it}, z_{it}) + \frac{1}{1+r} \mathbb{E}_t [v(z_{it+1}, (1-\delta) k_{it} + i_{it})]$$

Adjustment costs

$$c(i_{it}, k_{it}, z_{it}) = \underbrace{\frac{\gamma}{2} \left(\frac{i_{it}}{k_{it}} \right)^2 k_{it}}_{\text{convex}} + \underbrace{\mathbb{1}\left(i_{it} \neq 0\right) \left(F k_{it} + \lambda e^{z_{it}} k_{it}^{\alpha}\right)}_{\text{nonconvex}}$$

Irreversibilities

$$p(i_{it}) = \underbrace{1 \times \mathbb{1}\left(i_{it} \geq 0\right)}_{\text{buying}} + \underbrace{p_s \times \mathbb{1}\left(i_{it} < 0\right)}_{\text{selling}}$$

No Adjustment Costs

$$v(z_{it}, k_{it}) = \max_{i_{it}} e^{z_{it}} k_{it}^{\alpha} - i_{it} + \frac{1}{1+r} \mathbb{E}_t[v(z_{it+1}, (1-\delta)k_{it} + i_{it})]$$

No Adjustment Costs

$$v(z_{it}, k_{it}) = \max_{i_{it}} e^{z_{it}} k_{it}^{\alpha} - i_{it} + \frac{1}{1+r} \mathbb{E}_t[v(z_{it+1}, (1-\delta)k_{it} + i_{it})]$$

Optimal Behavior

$$1 = \frac{1}{1+r} \mathbb{E}_t[v_2(z_{it+1}, k_{it+1})]$$

$\rightarrow$ user cost model: $r + \delta = \mathbb{E}_t[\alpha k_{it+1}^{\alpha-1}]$

Convex Costs Only

$$v(z_{it}, k_{it}) = \max_{i_{it}} e^{z_{it}} k_{it}^{\alpha} - i_{it} - c(i_{it}, k_{it}, z_{it}) + \frac{1}{1+r}\mathbb{E}_t[v(z_{it+1}, k_{it+1})]$$

$$c(i_{it}, k_{it}, z_{it}) = \frac{\gamma}{2}\left(\frac{i_{it}}{k_{it}}\right)^2 k_{it}$$

Convex Costs Only

$$v(z_{it}, k_{it}) = \max_{i_{it}} e^{z_{it}} k_{it}^{\alpha} - i_{it} - c(i_{it}, k_{it}, z_{it}) + \frac{1}{1+r}\mathbb{E}_t[v(z_{it+1}, k_{it+1})]$$

$$c(i_{it}, k_{it}, z_{it}) = \frac{\gamma}{2}\left(\frac{i_{it}}{k_{it}}\right)^2 k_{it}$$

Optimal Behavior

$$1 + \gamma\left(\frac{i_{it}}{k_{it}}\right) = \frac{1}{1+r}\mathbb{E}_t[v_2(z_{it+1}, k_{it+1})]$$

$\rightarrow$ Q-theory model: $\dfrac{i_{it}}{k_{it}} = \dfrac{1}{\gamma}\left(\mathbb{E}_t[v_2(z_{it+1}, k_{it+1})] - 1\right)$

Nonconvex Costs

$$v(z_{it}, k_{it}) = \max_{i_{it}} e^{z_{it}} k_{it}^{\alpha} - i_{it} - c(i_{it}, k_{it}, z_{it}) + \frac{1}{1+r} \mathbb{E}_t[v(z_{it+1}, k_{it+1})]$$

$$c(i_{it}, k_{it}, z_{it}) = \frac{\gamma}{2} \left(\frac{i_{it}}{k_{it}} \right)^2 k_{it} + \mathbb{1} \left(i_{it} \neq 0 \right) \left(F k_{it} + \lambda e^{z_{it}} k_{it}^{\alpha} \right)$$

Nonconvex Costs

$$v(z_{it}, k_{it}) = \max_{i_{it}} e^{z_{it}} k_{it}^{\alpha} - i_{it} - c(i_{it}, k_{it}, z_{it}) + \frac{1}{1+r} \mathbb{E}_t [v(z_{it+1}, k_{it+1})]$$

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Optimal Behavior

$$v^{\mathrm{a}}(z_{it}, k_{it}) = \max_{i_{it}} e^{z_{it}} k_{it}^{\alpha} - i_{it} - c(i_{it}, k_{it}, z_{it}) + \frac{1}{1+r} \mathbb{E}_t [v(z_{it+1}, k_{it+1})]$$

$$v^{\mathrm{n}}(z_{it}, k_{it}) = e^{z_{it}} k_{it}^{\alpha} - i_{it} + \frac{1}{1+r} \mathbb{E}_t [v(z_{it+1}, (1-\delta) k_{it})]$$

Nonconvex Costs

$$v(z_{it}, k_{it}) = \max_{i_{it}} e^{z_{it}} k_{it}^{\alpha} - i_{it} - c(i_{it}, k_{it}, z_{it}) + \frac{1}{1+r}\mathbb{E}_t[v(z_{it+1}, k_{it+1})]$$

$$c(i_{it}, k_{it}, z_{it}) = \frac{\gamma}{2}\left(\frac{i_{it}}{k_{it}}\right)^2 k_{it} + \mathbb{1}\left(i_{it} \neq 0\right)\left(Fk_{it} + \lambda e^{z_{it}} k_{it}^{\alpha}\right)$$

Optimal Behavior

$$v^{a}(z_{it}, k_{it}) = \max_{i_{it}} e^{z_{it}} k_{it}^{\alpha} - i_{it} - c(i_{it}, k_{it}, z_{it}) + \frac{1}{1+r}\mathbb{E}_t[v(z_{it+1}, k_{it+1})]$$

$$v^{n}(z_{it}, k_{it}) = e^{z_{it}} k_{it}^{\alpha} - i_{it} + \frac{1}{1+r}\mathbb{E}_t[v(z_{it+1}, (1-\delta)k_{it})]$$

$\rightarrow$ Adjust iff $v^{a}(z_{it}, k_{it}) > v^{n}(z_{it}, k_{it})$

- Depreciation
- Productivity shock

Irreversibility

$$
\begin{aligned}
v(z_{it}, k_{it}) &= \max_{i_{it}} e^{z_{it}} k_{it}^{\alpha} - i_{it} + \frac{1}{1+r} \mathbb{E}_t[v(z_{it+1}, k_{it+1})] \\
p(i_{it}) &= 1 \times \mathbb{1}\left(i_{it} \geq 0\right) + p_s \times \mathbb{1}\left(i_{it} < 0\right)
\end{aligned}
$$

Irreversibility

$$
\begin{aligned}
v(z_{it}, k_{it}) &= \max_{i_{it}} e^{z_{it}} k_{it}^{\alpha} - i_{it} + \frac{1}{1+r} \mathbb{E}_t [v(z_{it+1}, k_{it+1})] \\
p(i_{it}) &= 1 \times \mathbb{1}\left(i_{it} \geq 0\right) + p_s \times \mathbb{1}\left(i_{it} < 0\right)
\end{aligned}
$$

Optimal Behavior

$$
\begin{aligned}
v^{\mathrm{b}}(z_{it}, k_{it}) &= \max_{i_{it} > 0} e^{z_{it}} k_{it}^{\alpha} - i_{it} + \frac{1}{1+r} \mathbb{E}_t [v(z_{it+1}, k_{it+1})] \\
v^{\mathrm{s}}(z_{it}, k_{it}) &= \max_{i_{it} < 0} e^{z_{it}} k_{it}^{\alpha} - p_s i_{it} + \frac{1}{1+r} \mathbb{E}_t [v(z_{it+1}, k_{it+1})] \\
v^{\mathrm{n}}(z_{it}, k_{it}) &= e^{z_{it}} k_{it}^{\alpha} + \frac{1}{1+r} \mathbb{E}_t [v(z_{it+1}, (1-\delta) k_{it})]
\end{aligned}
$$

Irreversibility

$$
\begin{aligned}
v(z_{it}, k_{it}) &= \max_{i_{it}} e^{z_{it}} k_{it}^{\alpha} - i_{it} + \frac{1}{1+r}\mathbb{E}_t[v(z_{it+1}, k_{it+1})] \\
p(i_{it}) &= 1 \times \mathbb{1}\,(i_{it} \geq 0) + p_s \times \mathbb{1}\,(i_{it} < 0)
\end{aligned}
$$

Optimal Behavior

$$
\begin{aligned}
v^{\mathrm{b}}(z_{it}, k_{it}) &= \max_{i_{it}>0} e^{z_{it}} k_{it}^{\alpha} - i_{it} + \frac{1}{1+r}\mathbb{E}_t[v(z_{it+1}, k_{it+1})] \\
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v^{\mathrm{n}}(z_{it}, k_{it}) &= e^{z_{it}} k_{it}^{\alpha} + \frac{1}{1+r}\mathbb{E}_t[v(z_{it+1}, (1-\delta)k_{it})]
\end{aligned}
$$

$\rightarrow$ Also generates inaction

Illustration of Various Frictions

Moment	LRD	No AC	CON	NC-F	NC-λ	TRAN
Fraction of inaction	0.081	0.0	0.038	0.616	0.588	0.69
Fraction with positive investment bursts	0.18	0.298	0.075	0.212	0.213	0.120
Fraction with negative investment bursts	0.018	0.203	0.0	0.172	0.198	0.024
corr(i_{it},i_{it-1})	0.058	-0.053	0.732	-0.057	-0.06	0.110
corr(i_{it}, a_{it})	0.143	0.202	0.692	0.184	0.196	0.346

Model Quantification

Overall strategy

1. Fix a subset of parameters
2. Estimate shock process using measured TFP-type approach
3. Estimate adjustment costs to match moments

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Fixed parameters

- Depreciation rate $\delta = 6.9\%$
- Discount rate $r = 5.25\%$

Model Quantification

Overall strategy

1. Fix a subset of parameters
2. Estimate shock process using measured TFP-type approach
3. Estimate adjustment costs to match moments

Fixed parameters

- Depreciation rate $\delta = 6.9\%$
- Discount rate $r = 5.25\%$

Estimate idiosyncratic shocks

- Assume $z_{it} = \varepsilon_{it} + b_t$
- Assume AR(1) and use GMM on

$$\log(\pi_{it}) = \rho_\epsilon \log(\pi_{it-1}) + \theta k_{it} - \rho_\epsilon \theta k_{it-1} + b_t - \rho_\epsilon b_{t-1} + \eta_{it}$$

- See paper for details

Estimating Adjustment Cost Parameters

- Estimate parameters for two separate cases:
  1. Fixed cost case: estimate $\Theta = (\gamma, F, p_s)$, set $\lambda = 1$
  2. Opportunity cost case: estimate $\Theta = (\gamma, \lambda, p_s)$, set $F = 0$

Estimating Adjustment Cost Parameters

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- **Simulated Method of Moments (SMM)**

$$\hat{\Theta} = \arg\min_{\Theta} \left[\Psi_\mathrm{d} - \Psi_\mathrm{s}(\Theta)\right]^T W \left[\Psi_\mathrm{d} - \Psi_\mathrm{s}(\Theta)\right]$$

  - Data moments Ψ_d: drawn from data
  - Model moments $\Psi_\mathrm{s}(\Theta)$: simulated panel of firms from model

  - Weighting matrix W: efficient matrix from GMM
  - Standard errors: GMM formulas plus factor for Monte Carlo error

Estimation Results: Fixed Cost Case

Spec.	Structural Parm. Est. (s.e.)			moments				
	γ	F	p_s	$corr(i, i_{-1})$	$corr(i, a)$	$spike^+$	$spike^-$	$\pounds(\hat{\Theta})$
LRD				0.058	0.143	0.186	0.018	
all	0.049 (0.002)	0.039 (0.001)	0.975 (0.004)	0.086	0.31	0.127	0.030	6399.9
γ only	0.455 (0.002)	0	1	0.605	0.540	0.23	0.028	53182.6
p_s only	0	0	0.795 (0.002)	0.113	0.338	0.132	0.033	7673.68
F only	0	0.0695 (0.00046)	1	-0.004	0.213	0.105	0.0325	7390.84

Estimation Results: Fixed Cost Case

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F only	0	0.0695 (0.00046)	1	-0.004	0.213	0.105	0.0325	7390.84

Estimated fixed cost $F \approxeq 4\%$ of capital stock

Estimation Results: Disruption Cost Case

Spec.	Structural Parm. Est. (s.e.)			moments				
	γ	λ	p_s	$corr(i, i_{-1})$	$corr(i, a)$	$spike^{+}$	$spike^{-}$	$\pounds(\hat{\Theta})$
LRD				0.058	0.143	0.186	0.018	
λ only	0	0.796 (0.0040)	1.0	-0.009	0.06	0.107	0.042	9384.06
all	0.153 (0.0056)	0.796 (0.0090)	0.981 (0.0090)	0.148	0.156	0.132	0.023	2730.97

Estimation Results: Disruption Cost Case

Spec.	Structural Parm. Est. (s.e.)			moments				
	γ	λ	p_s	$corr(i, i_{-1})$	$corr(i, a)$	$spike^+$	$spike^-$	$\pounds(\hat{\Theta})$
LRD				0.058	0.143	0.186	0.018	
λ only	0	0.796 (0.0040)	1.0	-0.009	0.06	0.107	0.042	9384.06
all	0.153 (0.0056)	0.796 (0.0090)	0.981 (0.0090)	0.148	0.156	0.132	0.023	2730.97

Estimated disruption cost $1 - \lambda \approxeq 20\%$ of profits

On average, pay 3.1% of profits in AC when adjust

Cooper and Haltiwanger (2006): Wrapping Up

- What types of adjustment costs do we need to match micro data?

Cooper and Haltiwanger (2006): Wrapping Up

- What types of adjustment costs do we need to match micro data? Non-convexities:
    - Fixed costs
    - Disruption costs
    - Irreversibilities

Cooper and Haltiwanger (2006): Wrapping Up

- What types of adjustment costs do we need to match micro data? Non-convexities:
    - Fixed costs
    - Disruption costs
    - Irreversibilities

- Nice illustration of Simulated Method of Moments (SMM) methodology
    - Specify moments of the data you think are important
    - Select parameters which are well-identified by those moments
    - Choose parameters to get model as close as possible to data

Asker, Collard-Wexler, and De Loecker (2014)

Shows Cooper-Haltiwanger (2006) model also explains much of MRPK_{it} dispersion documented by Hsieh and Klenow (2009)

Asker, Collard-Wexler, and De Loecker (2014)

Shows Cooper-Haltiwanger (2006) model also explains much of MRPK_{it} dispersion documented by Hsieh and Klenow (2009)

Data: LRD, 1972 - 1997

- Also use cross-country data for analysis in paper

Model: Cooper-Haltiwanger (2006) opportunity cost model

$$v(z_{it}, k_{it}) = \max_{i_{it}} e^{z_{it}} k_{it}^{\alpha} - i_{it} - c(i_{it}, k_{it}, z_{it}) + \frac{1}{1+r} \mathbb{E}_t[v(z_{it+1}, k_{it+1})]$$

$$c(i_{it}, k_{it}, z_{it}) = \frac{\gamma}{2} \left(\frac{i_{it}}{k_{it}} \right)^2 k_{it} + \mathbb{1}\left(i_{it} \neq 0\right) \lambda e^{z_{it}} k_{it}^{\alpha}$$

Estimation

Estimate $\Theta = (\gamma, \lambda)$ **using SMM**

$$\hat{\Theta} = \arg\min_{\Theta} \left[\Psi_{\mathrm{d}} - \Psi_{\mathrm{s}}(\Theta)\right]^{T} W \left[\Psi_{\mathrm{d}} - \Psi_{\mathrm{s}}(\Theta)\right]$$

Estimation

Estimate $\Theta = (\gamma, \lambda)$ using SMM

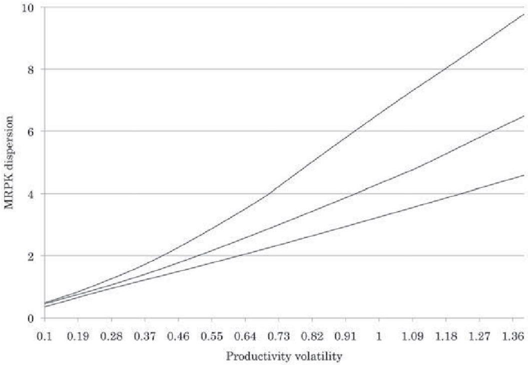
$$\hat{\Theta} = \arg\min_{\Theta} \left[\Psi_d - \Psi_s(\Theta)\right]^T W \left[\Psi_d - \Psi_s(\Theta)\right]$$

ADJUSTMENT COST ESTIMATES AND MOMENTS BY COUNTRY

	Adjustment Costs		Data Moments on Change in Log Capital		
Country	Convex	Fixed	Less than 5%	More than 20%	Standard Deviation
United States	8.80	.09	.39	.09	.21
Chile	4.10	.07	.19	.11	.28
India	3.46	.12	.29	.19	.30
France	.21	.00	.13	.57	.57
Spain	.74	.00	.20	.41	.59
Mexico	1.15	.22	.08	.73	.66
Romania	.66	.03	.08	.61	.72
Slovenia	.35	.00	.15	.52	.76

NOTE.—Standard errors were computed using the usual formula for minimum-distance estimators. However, because of the large size of the data sets we employ, the standard errors are of the order of 1×10^{-3} or smaller, and so we do not report them. Adjustment costs for Slovenia are based on a model with production function coefficients set to the mean US coefficients (see the discussion in Sec. V.B).

Higher Idiosyncratic Volatility $\rightarrow$ Higher MRPK Dispersion



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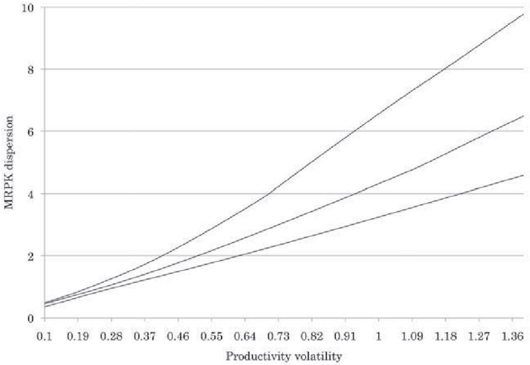

- $\text{MRPK}_{it} = \alpha \frac{y_{it}}{k_{it}}$
- Time to build → ex-post dispersion
- Adjustment costs → ex-ante dispersion

Idiosyncratic Volatility and MRPK Dispersion in Data

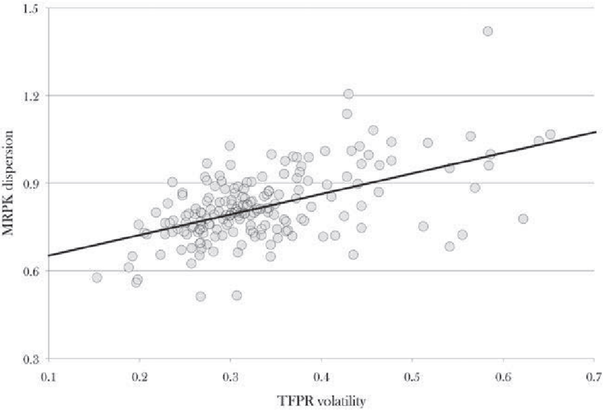

FIG. 2.—Volatility and the dispersion in MRPK: US plant data, 1972–97. The unit of observation is the industry. The line is generated by an OLS regression on 188 industries, in which the estimated slope is 0.73 (0.08) and the constant is 0.57 (0.03), and the $R^2 = .3$, where the standard errors are in parentheses.

Idiosyncratic Volatility and MRPK Dispersion in Data

Country	Coefficient	R^2	Industry-Year Observations
United States:			
Plants	.76***	.47	4,037
	(.04)		
Firms	.68***	.44	4,037
	(.07)		
Chile	.54*	.13	55
	(.29)		
France	1.03***	.28	167
	(.33)		
India	.61**	.28	279
	(.17)		
Mexico	.19**	.07	296
	(.07)		
Romania	.44***	.21	126
	(.13)		
Slovenia	.53**	.09	108
	(.21)		
Spain	.56*	.35	181
	(.33)		
All:			
Unweighted	.55***	.67	5,326
	(.15)		
Weighted	.74***	.50	5,326
	(.03)		

Quantitative Amount of Dispersion Explained

Country	Specification (1)	(2)	(3)	(4)	(5)
United States	.223	.806	.806	.643	.820
France	.892	.702	.899	.944	.651
Chile	.994	.983	.987	.963	.785
India	.984	.941	.964	.976	.596
Mexico	.879	.813	.883	.908	.634
Romania	.983	.923	.817	.702	.846
Slovenia	.967	.774	.967	.984	.683
Spain	.718	.627	.600	.530	.495
All (excluding United States)	.879	.777	.820	.800	.640
All	.674	.786	.816	.748	.696
Specification details:					
All US adjusted costs	X		X		
Own-country adjusted costs		X			
All 2 × US adjusted costs				X	
1-period time to build only					X
US average β's	X				
Industry-country β's		X	X	X	X

NOTE.—The unit of observation is the country-industry. Specifications are as follows: (1) All countries have the United States' estimated adjustment costs and production coefficients equal to the US averages across industries; (2) industry-country-specific production coefficients (except for Slovenia; see Sec. III.B), country specific adjustment costs, and industry-country-specific AR(1); (3) same as for 2, but with the United States' estimated adjustment costs for all countries; (4) same as for 3, but with twice the United States' estimated adjustment costs for all countries; and (5) same as for 3, but with zero adjustment costs (other than the one-period time to build) for all countries. In all specifications, the

Plan for this Topic

1. An unfair summary of the empirical investment literature

2. Accounting for micro-level investment behavior with nonconvex adjustment costs

3. **Macro implications of nonconvex adjustment costs**

Aggregate Implications of Micro Investment Models

1. Aggregation of micro-level models holding prices fixed (partial equilibrium)
    - Response of aggregate investment to shocks depends on number of firms who adjust
    - Aggregate investment features time-varying elasticity w.r.t. shocks
    - Representative firm instead predicts constant elasticity

Aggregate Implications of Micro Investment Models

1. Aggregation of micro-level models holding prices fixed (partial equilibrium)
    - Response of aggregate investment to shocks depends on number of firms who adjust
    - Aggregate investment features time-varying elasticity w.r.t. shocks
    - Representative firm instead predicts constant elasticity

2. Endogenize prices in general equilibrium
    - In benchmark RBC framework, procyclical real interest rate eliminates time-varying elasticity
    - Modifications to benchmark model can break this irrelevance result

General Lessons

1. Anytime you go from micro to macro, need to think about
    - Aggregation
    - General equilibrium

2. Macro models with micro heterogeneity are hard
    - Entire cross-sectional distribution of agents part of state vector
    - Difficult to numerically compute and estimate

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    - General equilibrium

2. Macro models with micro heterogeneity are hard
    - Entire cross-sectional distribution of agents part of state vector
    - Difficult to numerically compute and estimate

- Aggregate implications of lumpy investment models good illustration of these more general issues
    - Each of these steps has been extensively studied

Outline of Next Steps

1. Benchmark general equilibrium model with lumpy investment: Khan and Thomas (2008)
   - Aside: how to numerically compute heterogeneous agent models

2. Model generates time-varying elasticity in partial equilibrium

3. Model generates constant elasticity in general equilibrium

4. Two broad responses to irrelevance result in literature
   - Specification of micro-level adjustment costs
   - Specification of general equilibrium

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Model Overview

Heterogeneous Firms

- Fixed mass
- Idiosyncratic + aggregate productivity shocks
- Fixed capital adjustment costs

Representative Household

- Owns firms
- Supplies labor
- Complete markets

Heterogeneous Firms

Production technology $y_{jt} = e^{z_t} e^{\varepsilon_{jt}} k_{jt}^{\theta} n_{jt}^{\nu}, \theta + \nu < 1$

- Idiosyncratic productivity shock $\varepsilon_{jt+1} = \rho_\epsilon \varepsilon_{jt} + \omega^{\varepsilon}_{jt+1}$ where $\omega^{\varepsilon}_{jt+1} \sim N(0, \sigma^2_\epsilon)$
- Aggregate productivity shock $z_{t+1} = \rho_z z_t + \omega^{z}_{t+1}$ where $\omega^{z}_{t+1} \sim N(0, \sigma^2_z)$

Heterogeneous Firms

Production technology $y_{jt} = e^{z_t} e^{\varepsilon_{jt}} k_{jt}^{\theta} n_{jt}^{\nu}$, $\theta + \nu < 1$

- Idiosyncratic productivity shock $\varepsilon_{jt+1} = \rho_{\epsilon} \varepsilon_{jt} + \omega_{jt+1}^{\varepsilon}$ where $\omega_{jt+1}^{\varepsilon} \sim N(0, \sigma_{\epsilon}^2)$
- Aggregate productivity shock $z_{t+1} = \rho_z z_t + \omega_{t+1}^{z}$ where $\omega_{t+1}^{z} \sim N(0, \sigma_z^2)$

Firms accumulate capital according to $k_{jt+1} = (1-\delta) k_{jt} + i_{jt}$

- If $\frac{i_{jt}}{k_{jt}} \notin [-a, a]$, pay fixed cost ξ_{jt} in units of labor
- Fixed cost $\xi_{jt} \sim U[0, \overline{\xi}]$

Firm Optimization Problem: Recursive Formulation

$$
\begin{aligned}
v(\varepsilon, k, \xi; \mathbf{s}) = \max_n \; & e^{z} e^{\varepsilon} k^{\theta} n^{\nu} - w(\mathbf{s})\, n \\
& + \max\left\{ v^{A}(\varepsilon, k; \mathbf{s}) - w(\mathbf{s})\,\xi,\; v^{N}(\varepsilon, k; \mathbf{s}) \right\}
\end{aligned}
$$

Firm Optimization Problem: Recursive Formulation

$$
\begin{aligned}
v(\varepsilon, k, \xi; \mathbf{s}) = \max_n \, & e^z e^\varepsilon k^\theta n^\nu - w(\mathbf{s}) n \\
& + \max \left\{ v^A(\varepsilon, k; \mathbf{s}) - w(\mathbf{s}) \xi, v^N(\varepsilon, k; \mathbf{s}) \right\}
\end{aligned}
$$

$$
v^A(\varepsilon, k; \mathbf{s}) = \max_{i \in \mathbb{R}} -i + \mathbb{E}\left[\Lambda(\mathbf{s}') v(\varepsilon', k', \xi'; \mathbf{s}') | \varepsilon, k; \mathbf{s}\right]
$$

$$
v^N(\varepsilon, k; \mathbf{s}) = \max_{i \in [-ak, ak]} -i + \mathbb{E}\left[\Lambda(\mathbf{s}') v(\varepsilon', k', \xi'; \mathbf{s}') | \varepsilon, k; \mathbf{s}\right]
$$

Firm Optimization Problem: Recursive Formulation

$$v(\varepsilon, k, \xi; \mathbf{s}) = \max_{n} e^{z} e^{\varepsilon} k^{\theta} n^{\nu} - w(\mathbf{s})\, n + \max \left\{ v^{A}(\varepsilon, k; \mathbf{s}) - w(\mathbf{s})\, \xi,\; v^{N}(\varepsilon, k; \mathbf{s}) \right\}$$

$$\widehat{v}(\varepsilon, k; \mathbf{s}) = \max_{n} e^{z} e^{\varepsilon} k^{\theta} n^{\nu} - w(\mathbf{s})\, n + \frac{\widehat{\xi}(\varepsilon, k; \mathbf{s})}{\overline{\xi}} \left(v^{A}(\varepsilon, k; \mathbf{s}) - w(\mathbf{s}) \frac{\widehat{\xi}(\varepsilon, k; \mathbf{s})}{2} \right) + \left(1 - \frac{\widehat{\xi}(\varepsilon, k; \mathbf{s})}{\overline{\xi}} \right) v^{N}(\varepsilon, k; \mathbf{s})$$

Household

Representative household who owns all firms in the economy

$$\max_{C_t, N_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - aN_t\right) \text{ such that}$$

$$C_t = w_t N_t + \Pi_t$$

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Complete markets implies that $\Lambda_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-1}$

- Firms maximize their market value
- Market value given by expected present value of dividends using stochastic discount factor
- With complete markets, SDF is household's intertemporal marginal rate of substitution

Defining Recursive Competitive Equilibrium

What is the aggregate state $\mathbf{s}$?

Defining Recursive Competitive Equilibrium

What is the aggregate state **s**?

- Aggregate shock z

Defining Recursive Competitive Equilibrium

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- Firm's individual states: productivity ε and capital k

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What is the law of motion for the **s**?

$$g_{t+1}(\varepsilon', k') = \int \begin{bmatrix} 1\{\rho_\varepsilon \varepsilon + \sigma_\varepsilon \omega'_\varepsilon = \varepsilon'\} \\ \times \int 1\{k'_t(\varepsilon, k, \xi) = k'\}\, dG(\xi) \end{bmatrix} \times p(\omega'_\varepsilon)\, g_t(\varepsilon, k)\, d\omega'_\varepsilon d\varepsilon dk$$

Recursive Competitive Equilibrium

A set of $v(\varepsilon, k; z, g)$, $C(z, g)$, $w(z, g)$, $\Lambda(z'; z, g)$, and $g'(z, g)$ such that

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2. **Household optimization**: $w(z, g)C(z, g)^{-1} = a$

3. **Market clearing + consistency**:

$$\Lambda(z'; z, g) = \beta \left(\frac{C(z', g'(z, g))}{C(z, g)} \right)^{-1}$$

$$C(z, g) = \int (y(\varepsilon, k, \xi; z, g) - i(\varepsilon, k, \xi; z, g)) dG(\xi) g(\varepsilon, k) d\varepsilon dk$$

$g'(\varepsilon, k)$ satisfies law of motion for distribution

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- Today will give you an overview to help you read papers
  - My HW2: solve steady state
  - Aggregate dynamics: Khan and Thomas (2008); Winberry (2016); Terry (2016)

Steady State Recursive Competitive Equilibrium

A set of $v^*(\varepsilon, k)$, C^*, w^*, and $g^*(\varepsilon, k)$ such that

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Hopenhayn-Rogerson (1993) Algorithm

Start with guess of w^*

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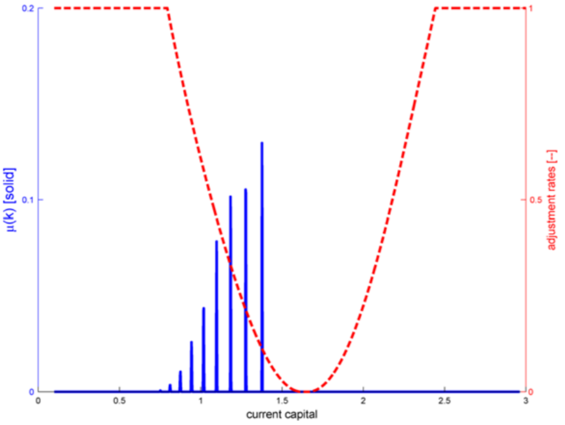
Start with guess of w^*

- Solve firm optimization problem $\rightarrow v^*(\varepsilon, k)$
- Compute stationary distribution $g^*(\varepsilon, k)$
- Compute implied aggregate consumption C^*
- Check household optimization $w^*(C^*)^{-1} = a$

Update guess of w^*

Steady State Outcomes



Distribution in model with no idiosyncratic productivity shocks

Investment decision characterized by adjustment hazard

Full Model with Aggregate Shocks

- Outside of steady state, three key challenges

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  1. Distribution g varies over time $\rightarrow$ how to approximate distribution?
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- If curious: continuous time makes this easier (Ahn, Kaplan, Moll,

Krusell and Smith (1998)

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Krusell and Smith (1998)

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- **Given guess $\boldsymbol{\alpha}$ and $\boldsymbol{\gamma}$**
  - Compute individual decisions $v(\varepsilon, k; z, \overline{K})$
  - Simulate decision rules $\rightarrow \{\overline{K}_t, C_t, z_t\}$
- **Update $\boldsymbol{\alpha}$ and $\boldsymbol{\gamma}$ using OLS**

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- R^2 on regressions typical accuracy measure
  - Only $\overline{K}$ matters $\rightarrow$ distribution not important ("approximate aggregation")
  - Problems with this measure: Den Haan (2010)

Winberry (2018)

- Approximate distribution with parametric family:

$$g\left(\varepsilon,k\right)\cong g_0\,\mathrm{exp}\{g_1^1\left(\varepsilon-m_1^1\right)+g_1^2\left(k-m_1^2\right)+$$
$$\sum_{i=2}^{n_g}\sum_{j=0}^{i}g_i^j\left[\left(\varepsilon-m_1^1\right)^{i-j}\left(k-m_1^2\right)^j-m_i^j\right]\}$$

$\rightarrow$ Aggregate state approximated by $(z, g(\varepsilon, k)) \approx (z, \mathbf{m})$

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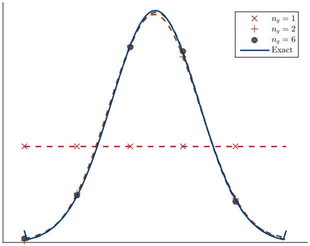
$\rightarrow$ Aggregate state approximated by $(z, g(\varepsilon, k)) \approx (z, \mathbf{m})$

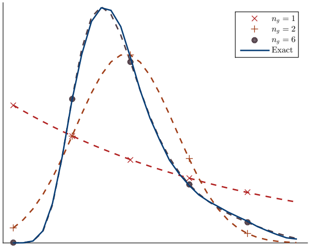
- Compute law of motion + prices directly by integration
- Compute aggregate dynamics using perturbation methods
  - Solve for steady state in `Matlab`
  - Solve for aggregate dynamics using `Dynare`

Winberry (2018)

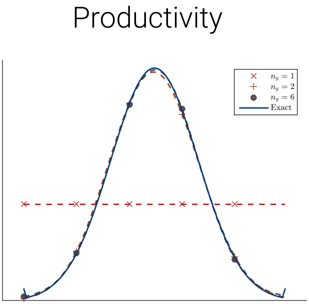
Productivity

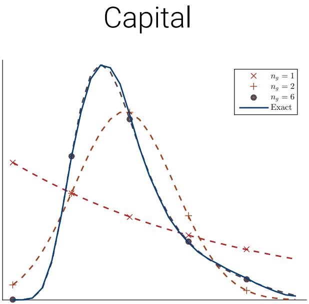


Capital



Winberry (2018)





- Run time $\approx$ 20 - 40 seconds for accurate approximation
- Fast enough for likelihood-based estimation
- Codes at my `website`

Outline of Next Steps

1. Benchmark general equilibrium model with lumpy investment: Khan and Thomas (2008)
   - Aside: how to numerically compute heterogeneous agent models

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4. Two broad responses to irrelevance result in literature
   - Specification of micro-level adjustment costs
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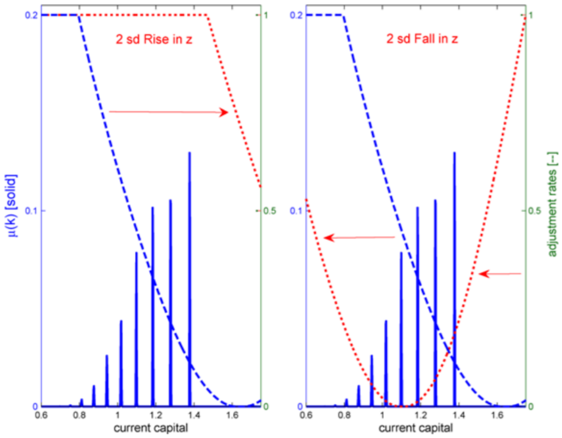
Khan and Thomas (2008) Calibration

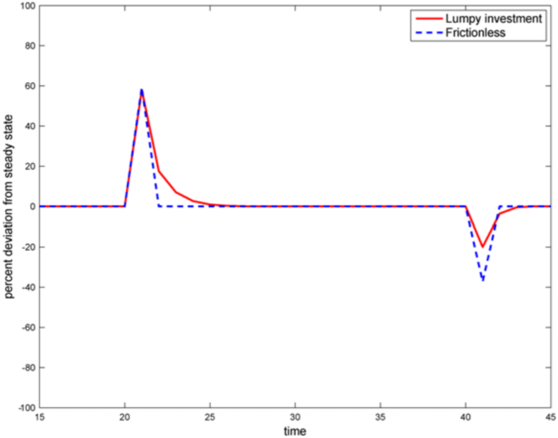
Parameter	Description	Value
Households		
β	Discount factor	.961
ψ	Labor disutility	$N^* = \frac{1}{3}$
Firms		
ν	Labor share	.64
θ	Capital share	.256
δ	Capital depreciation	.085
$\overline{\xi}$	Fixed cost	.0083
a	No fixed cost region	.011
ρ_ε	Idiosyncratic TFP AR(1)	.859
σ_ε	Idiosyncratic TFP AR(1)	.022
Aggregate shock		
ρ_Z	Aggregate TFP AR(1)	.859
σ_Z	Aggregate TFP AR(1)	.014

Complicated Impulse Responses



Response of aggregate investment to shock depends on interaction of initial distribution and adjustment hazards

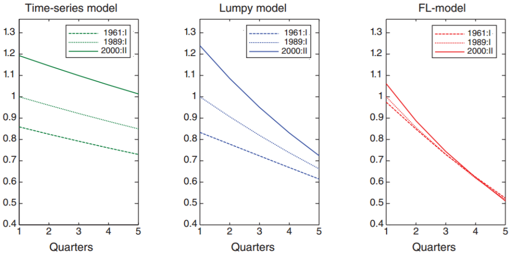
Implication: Sign Dependence



Aggregate investment more responsive to positive than negative shocks

Note true in frictionless model

Implication: State Dependence



From Bachmann, Caballero, and Engel (2013)

$$\frac{I_t}{K_t} = \sum_{j=1}^{p} \phi_j \frac{I_{t-j}}{K_{t-j}} + \sigma_t e_t$$

$$\sigma_t = \alpha_1 + \eta_1 \frac{1}{p} \sum_{j=1}^{p} \frac{I_{t-j}}{K_{t-j}}$$

Aggregate Nonlinearities

- Both of these are examples of nonlinear aggregate dynamics
    - Linear model has constant loading on aggregate shock

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- Some evidence in aggregate data
  - Sign and state dependence $\rightarrow$ distribution of $\frac{I_t}{K_t}$ positively skewed
  - State dependence $\rightarrow$ dynamics of $\frac{I_t}{K_t}$ feature conditional heteroskedasticity

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  - Sign and state dependence $\rightarrow$ distribution of $\frac{I_t}{K_t}$ positively skewed
  - State dependence $\rightarrow$ dynamics of $\frac{I_t}{K_t}$ feature conditional heteroskedasticity
- My view: time series evidence is suggestive at best
  - Predictions are about extreme states, which are rare
  - But that is exactly when we care about these predictions!

    $\rightarrow$ rely on cross-sectional data + carefully specified general equilibrium model

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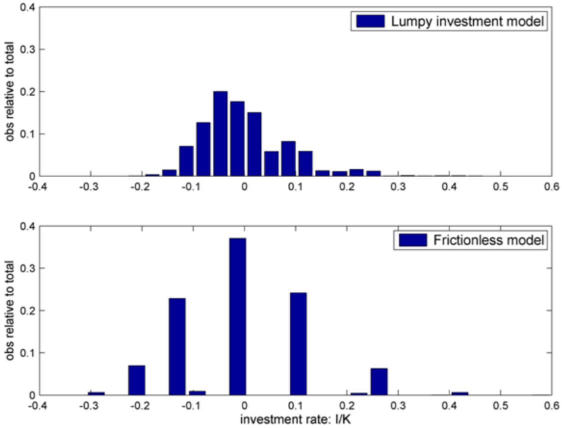
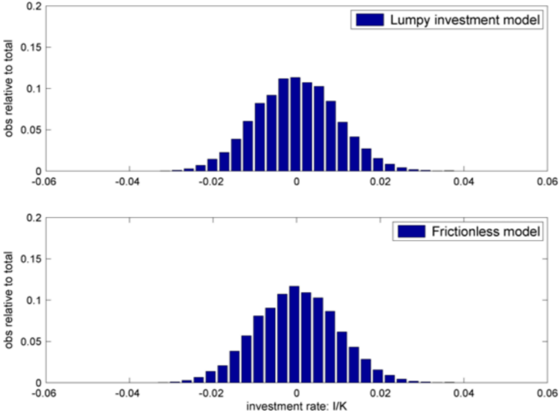
5. If time, discuss policy implications

Distribution of Aggregate $\frac{I_t}{K_t}$ in Partial Equilibrium



Distribution of Aggregate $\frac{I_t}{K_t}$ in General Equilibrium



Distribution of Aggregate $\frac{I_t}{K_t}$ in General Equilibrium

TABLE III

ROLE OF NONCONVEXITIES IN AGGREGATE INVESTMENT RATE DYNAMICS

	Persistence	Standard Deviation	Skewness	Excess Kurtosis
Postwar U.S. data[a]	*0.695*	*0.008*	*0.008*	*−0.715*
A. Partial equilibrium models				
PE frictionless	−0.069	0.128	0.358	0.140
PE lumpy investment	0.210	0.085	1.121	2.313
B. General equilibrium models				
GE frictionless	0.659	0.010	0.048	0.048
GE lumpy investment	0.662	0.010	0.067	−0.074

[a]Data are annual private investment-to-capital ratio, 1954–2005, computed using Bureau of Economic Analysis tables.

Business Cycles Nearly Identical to Representative Firm

TABLE IV

AGGREGATE BUSINESS CYCLE MOMENTS

	Output	TFP[a]	Hours	Consump.	Invest.	Capital
A. Standard deviations relative to output[b]						
GE frictionless	(2.277)	0.602	0.645	0.429	3.562	0.494
GE lumpy	(2.264)	0.605	0.639	0.433	3.539	0.492
B. Contemporaneous correlations with output						
GE frictionless		1.000	0.955	0.895	0.976	0.034
GE lumpy		1.000	0.956	0.900	0.976	0.034

[a] Total factor productivity.

[b] The logarithm of each series is Hodrick–Prescott-filtered using a weight of 100. The output column of panel A reports percent standard deviations of output in parentheses.

Why Do the Nonlinearities Disappear?

General equilibrium price movements

- Time-varying elasticity comes from large movements in adjustment hazard
- Procyclical real interest rate and wage restrain those movements

$$1 + r_t = \frac{1}{\mathbb{E}[\Lambda_{t,t+1}]}$$

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$$1 + r_t = \frac{1}{\mathbb{E}[\Lambda_{t,t+1}]}$$

Specification of adjustment costs

- Calibrated adjustment costs small

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   - Specification of general equilibrium

Bachmann, Caballero, and Engel (2013)

- Argue Khan and Thomas’ calibration of adjustment costs responsible for irrelevance result
- Calibrate larger adjustment costs and recover aggregate nonlinearities

Bachmann, Caballero, and Engel (2013)

- Argue Khan and Thomas' calibration of adjustment costs responsible for irrelevance result
- Calibrate larger adjustment costs and recover aggregate nonlinearities
- Argument based on decomposition between AC smoothing and PR smoothing
  - Frictionless partial equilibrium model excessively volatile
  - AC smoothing: dampening due to adjustment costs
  - PR smoothing: dampening due to price movements
- Measure AC smoothing in data and target in calibration $\rightarrow$ higher adjustment costs

Model

Production technology $y_{jt} = e^{z_t} e^{\varepsilon_{st}} e^{\varepsilon_{jt}} k_{jt}^{\theta} n_{jt}^{\nu}, \theta + \nu < 1$

- Idiosyncratic productivity shock $\varepsilon_{jt+1} = \rho_{\epsilon} \varepsilon_{jt} + \omega_{jt+1}^{\varepsilon}$ where $\omega_{jt+1}^{\varepsilon} \sim N(0, \sigma_{\epsilon}^2)$
- Aggregate productivity shock $z_{t+1} = \rho_z z_t + \omega_{t+1}^{z}$ where $\omega_{t+1}^{z} \sim N(0, \sigma_z^2)$
- Sectoral productivity shock $\varepsilon_{st+1} = \rho_{\epsilon} \varepsilon_{st} + \omega_{st+1}^{\varepsilon}$ where $\omega_{st+1}^{\varepsilon} \sim N(0, \sigma_{\epsilon_s}^2)$

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- Sectoral productivity shock $\varepsilon_{st+1} = \rho_\epsilon \varepsilon_{st} + \omega_{st+1}^{\varepsilon}$ where $\omega_{st+1}^{\varepsilon} \sim N(0, \sigma_{\epsilon_s}^2)$

Firms **accumulate capital** according to $k_{jt+1} = (1 - \delta) k_{jt} + i_{jt}$

- If don't pay fixed cost, must undertake maintenance investment $\chi \times \delta k_{jt}$
- Otherwise, pay fixed cost ξ_{jt} in units of labor
- Fixed cost $\xi_{jt} \sim U[0, \overline{\xi}]$

Calibration

Set most parameters exogenously

Choose σ_z, $\bar{\xi}$, and χ to match degree of AC-smoothing

- Identify AC-smoothing using volatility of sectoral investment rates
  - Aggregated enough to capture interaction of distribution and hazards
  - Small enough to not generate price response

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Set most parameters exogenously

Choose σ_z, $\overline{\xi}$, and χ to match degree of AC-smoothing

- Identify AC-smoothing using volatility of sectoral investment rates
  - Aggregated enough to capture interaction of distribution and hazards
  - Small enough to not generate price response
- Targets:
  1. Volatility of aggregate investment rate
  2. Average volatility of sectoral investment rates
  3. Amount of conditional heteroskedasticity

AC vs. PR Smoothing Decomposition

TABLE 6—SMOOTHING DECOMPOSITION

Model	AC smoothing/total smoothing (in percent)		
	LB	UB	Average
Khan-Thomas-lumpy annual	0.0	16.1	8.0
Khan-Thomas-lumpy annual, our $\overline{\xi}$	8.1	59.2	33.7
Our model annual ($\chi = 0$), Khan and Thomas' $\overline{\xi}$	0.8	16.0	8.4
Our model annual ($\chi = 0$)	18.9	75.3	47.0
Our model annual ($\chi = 0.25$)	19.1	75.7	47.4
Our model annual ($\chi = 0.50$)	19.9	76.6	48.3
Our model quarterly ($\chi = 0$)	14.5	80.9	47.7
Our model quarterly ($\chi = 0.25$)	15.4	80.9	48.2
Our model quarterly ($\chi = 0.5$)	15.4	81.0	48.2

$$UB = \log\left[\sigma(\text{none})/\sigma(\text{AC})\right] / \log\left[\sigma(\text{none})/\sigma(\text{both})\right]$$

$$LB = 1 - \log\left[\sigma(\text{none})/\sigma(\text{PR})\right] / \log\left[\sigma(\text{none})/\sigma(\text{both})\right]$$

Calibrated Adjustment Costs

TABLE 4—THE ECONOMIC MAGNITUDE OF ADJUSTMENT COSTS—ANNUAL

Model	Adjustment costs/ unit's output (in percent) (1)	Adjustment costs/ unit's wage bill (in percent) (2)
This paper ($\chi = 0$)	38.9	60.9
This paper ($\chi = 0.25$)	12.7	19.8
This paper ($\chi = 0.50$)	3.6	5.6
Caballero-Engel (1999)	16.5	—
Cooper-Haltiwanger (2006)	22.9	—
Bloom (2009)	35.4	—
Khan-Thomas (2008)	0.5	0.8
Khan-Thomas (2008) "Huge Adj. Costs"	3.7	5.8

Notes: This table displays the average adjustment costs paid, conditional on adjustment, as a fraction of output (left column) and as a fraction of the wage bill (right column), for various models. Rows 4–6 are based on table IV in Bloom (2009). For Cooper and Haltiwanger (2006) and Bloom (2009) we report the sum of costs associated with two sources of lumpy adjustment: fixed adjustment costs and partial irreversibility. The remaining models only have fixed adjustment costs.

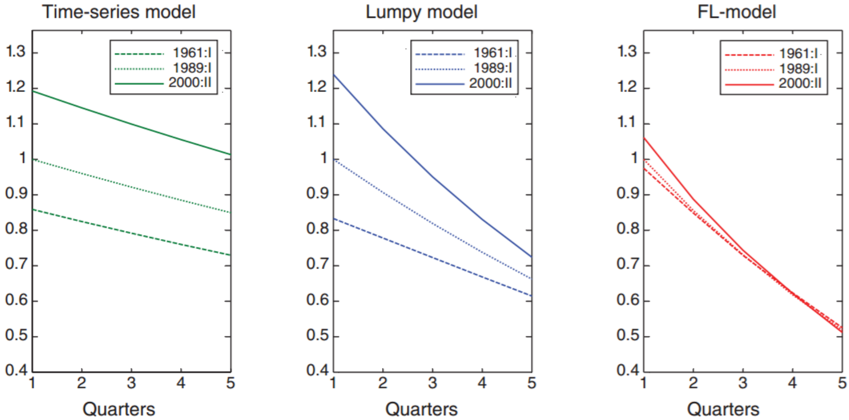
Aggregate Nonlinearities

TABLE 5—HETEROSCEDASTICITY RANGE

Model	$\log(\sigma_{95}/\sigma_5)$
Data	*0.3021*
This paper ($\chi = 0$)	0.1830
This paper ($\chi = 0.25$)	0.2173
This paper ($\chi = 0.50$)	0.2901
Quadratic adj. costs ($\chi = 0$)	0.0487
Quadratic adj. costs ($\chi = 0.25$)	0.0411
Quadratic adj. costs ($\chi = 0.50$)	0.0321
Frictionless	0.0539
Khan-Thomas (2008)	0.0468

Notes: This table displays heteroscedasticity range ($\log(\sigma_{95}/\sigma_5)$) for the data (row 1) and various model specifications that vary in terms of the maintenance parameter χ and the adjustment technology for capital: fixed adjustment costs (rows 2–4), quadratic adjustment costs (rows 5–7), a frictionless model, and the Khan-Thomas (2008) model. The adjustment costs for the models in rows 2–7 have been calibrated to match aggregate and sectoral investment rate volatilities.

Aggregate Nonlinearities



Aggregate Nonlinearities

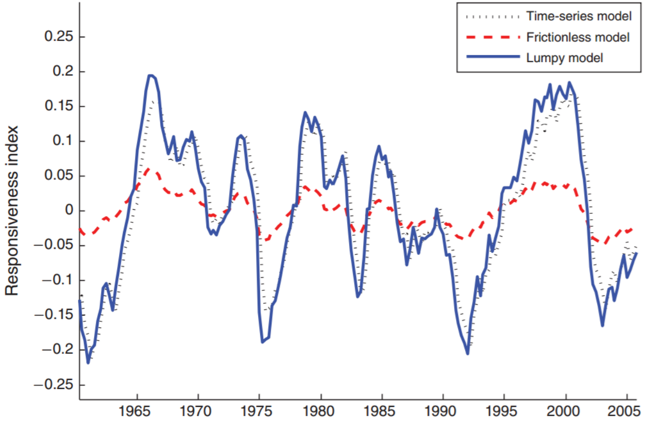


FIGURE 3. TIME PATHS OF THE RESPONSIVENESS INDEX

Aggregate Nonlinearities

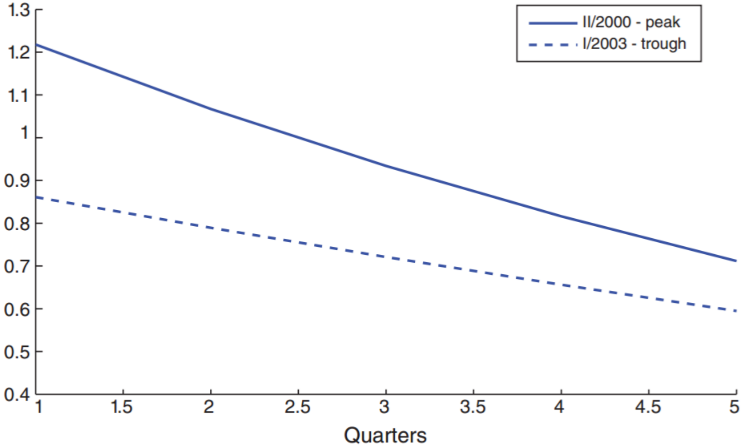


FIGURE 7. IMPULSE RESPONSES OF THE AGGREGATE INVESTMENT RATE IN THE 2000 BOOM-BUST CYCLE

Outline of Next Steps

1. Benchmark general equilibrium model with lumpy investment: Khan and Thomas (2008)
   - Aside: how to numerically compute heterogeneous agent models

2. Model generates time-varying elasticity in partial equilibrium

3. Model generates constant elasticity in general equilibrium

4. **Two broad responses to irrelevance result in literature**
   - Specification of micro-level adjustment costs
   - Specification of general equilibrium: Winberry (2018), Bachmann and Ma (2016), Cooper and Willis (2014)

Winberry (2018)

- Argues that procyclical interest rate in Khan and Thomas' model inconsistent with data
  - Cooper and Willis (2014): feed in from data
  - Winberry (2018): general equilibrium model
- When consistent with data recover aggregate nonlinearities

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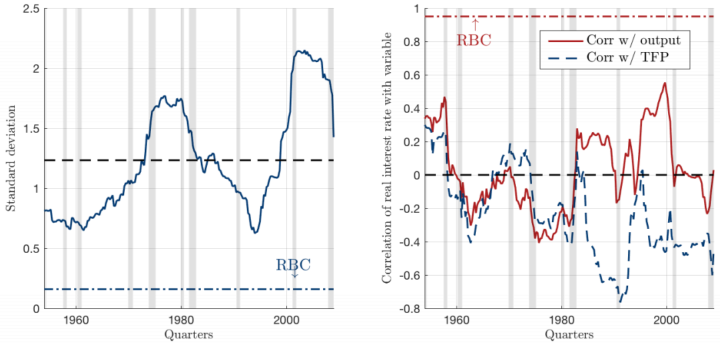
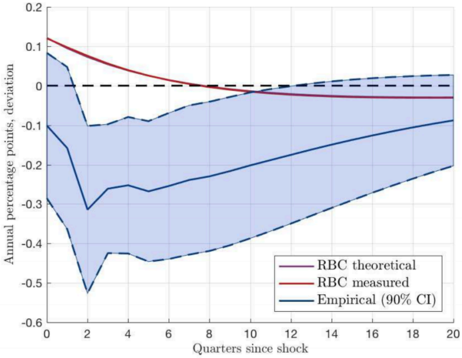
	$\boldsymbol{\sigma}(r_t)$	$\boldsymbol{\rho}(r_t, y_{t-1})$	$\boldsymbol{\rho}(r_t, y_t)$	$\boldsymbol{\rho}(r_t, y_{t+1})$
T-bill	2.18%	-0.08	-0.17	-0.251
AAA	2.34%	-0.29	-0.37	-0.40
BAA	2.43%	-0.32	-0.41	-0.45
Stock	24.7%	-0.24	-0.14	0.02
RBC	0.16%	0.61	0.97	0.74

Rolling Windows of r_t Dynamics



IRF of r_t to TFP Shock



Model

Firms as in Khan and Thomas except:

- Corporate tax code
- Temporary investment stimulus policy
- Quadratic adjustment costs

Model

Firms as in Khan and Thomas except:

- Corporate tax code
- Temporary investment stimulus policy
- Quadratic adjustment costs

Household preferences feature habit formation:

$$\max_{C_t, N_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log\left(C_t - H_t - \chi \frac{N_t^{1+\eta}}{1+\eta}\right)$$

$$S_t = \frac{C_t - H_t}{C_t} \text{ and } \log S_t = (1-\rho_S)\log \overline{S} + \rho_S \log S_{t-1} + \lambda \log \frac{C_t}{C_{t-1}}$$

Calibration

Set most parameters exogeneously

Calibration

Set most parameters exogeneously

Choose parameters governing micro heterogeneity and habit formation to match micro investment data and real interest rate dynamics

- Real interest rate dynamics pin down capital supply and demand curves
    - Capital supply: households smoothing consumption $\rightarrow$ habit formation
    - Capital demand: firms demanding future capital $\rightarrow$ shocks and adjustment costs
- Micro investment data pins down shocks and adjustment costs

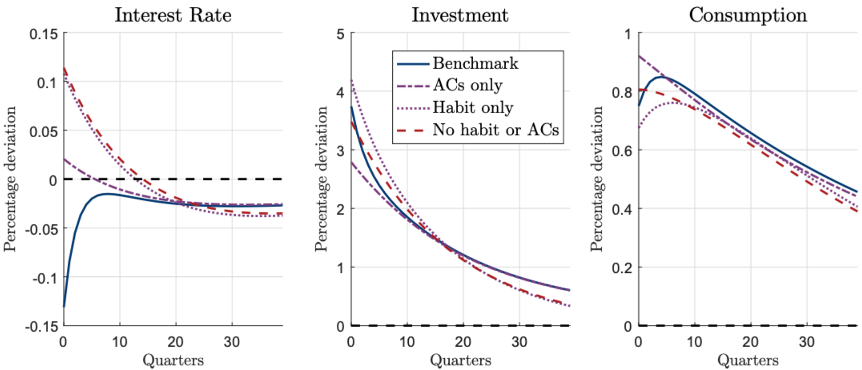
Calibration

TABLE 3
EMPIRICAL TARGETS

Micro Investment		
Target	**Data**	**Model**
Average investment rate (%)	10.4%	10.7%
Standard deviation of investment rates	0.16	0.15
Spike rate (%)	14.4%	19.0%
Positive investment rates (%)	85.6%	81.0%
Interest Rate Dynamics		
Target	**Data**	**Model**
Cumulative impulse response	−0.49	−0.31
$\sigma(I_t)/\sigma(Y_t)$	2.87	2.88

Calibration

FIGURE 3: Identification of Habit Formation and Adjustment Costs



Calibration

TABLE 4
FITTED PARAMETER VALUES

Micro Heterogeneity		
Parameter	**Description**	**Value**
$\bar{\xi}$	Upper bound on fixed costs	0.53
φ	Convex adjustment cost	2.34
ρ_ε	Idiosyncratic productivity AR(1) (*fixed*)	0.90
σ_ε	Idiosyncratic productivity AR(1)	0.056
Habit Formation		
Parameter	**Description**	**Value**
λ	Sensitivity of habit w.r.t. consumption bundle	0.73

State Dependence of TFP Shocks

TABLE 6
FLUCTUATIONS IN RESPONSIVENESS INDEX OVER TIME

	95-5 ratio	90-10 ratio	75-25 ratio	$\rho(RI_t, \log Y_t)$	$\rho(RI_t, \text{adj}_t)$
Benchmark Calibration (PE interest elasticity $d \log I_t / dr_t = -7.55$)					
Partial Equilibrium	64%	50%	25%	0.93	0.93
General Equilibrium	31%	23%	15%	0.99	0.78
Khan and Thomas (2008) Calibration (PE interest elasticity $d \log I_t / dr_t = -1055.41$)					
Partial Equilibrium	49%	38%	18%	0.92	0.94
General Equilibrium	7%	5%	3%	0.98	0.93

$$RI_t = 100 \times \log \left(\frac{I(z_t + \sigma_z, X_t, \mu_t) - I(z_t, X_t, \mu_t)}{I(\sigma_z, X^*, \mu^*) - I(0, X^*, \mu^*)} \right)$$

State Dependence of Stimulus Policy

TABLE 8
RESPONSIVENESS INDEX FOR INVESTMENT STIMULUS SHOCK

	95-5 ratio	90-10 ratio	75-25 ratio	$\rho(RI_t, \log Y_t)$	$\rho(RI_t, \text{adj}_t)$
Impact	22%	15%	6%	0.86	0.60
Cumulative	17%	11%	5%	0.78	0.66

$$\text{price of investment} = 1 - \text{sub}_t$$

Conclusion: Takeaways from Topic 2

1. **Investment is lumpy** in the microdata

2. Structural micro models provide evidence for **nonconvex adjustment costs**
   - SMM estimation

3. Calibrated macro models indicate possibly generates **time-varying aggregate elasticity**
   - Aggregation and general equilibrium both important
   - Solving models with distribution in state vector